1. Use the diagonal form method to compute $e^{At}$ for

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2. By direct differentiation of

$$x(t) = e^{At}x_0 + \int_0^t e^{A(t-\sigma)}Bu(\sigma) \, d\sigma$$

show that

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0$$

3. For the time-invariant, $n$-dimensional, nonlinear, single-input state equation

$$\dot{x}(t) = Ax(t) + Dx(t)u(t) + Bu(t), \quad x(0) = 0$$

Show that under appropriate additional hypotheses a solution is

$$x(t) = \int_0^t e^{A(t-\sigma)}\int_0^\sigma Bu(\tau)d\tau \, d\sigma$$

4. Use a power series approach to find a solution $X(t)$ for the $n\times n$ matrix differential equation

$$\dot{X}(t) = AX(t) + X(t)F, \quad X(0) = X_0$$

5. Using the result of Problem 4, what condition on $X_0$ will guarantee that the $n\times n$ matrix differential equation

$$\dot{X}(t) = AX(t) - X(t)A, \quad X(0) = X_0$$

has a constant solution?

**Instructions** In a graduate course such as this, homework should be an individual effort. On the other hand, students should be encouraged to discuss the course material and help each other with obscurities and difficulties. The following policy is an attempt to fairly delineate the boundaries of homework collaboration. *Discussion of particular aspects of the homework assignment is permitted for clarification of the problems, but no notes should be carried away from the discussion. The written work you hand in should be your own work.* Copying homework, using a pirated solutions manual, and using solutions supplied from other years or other universities are ethics violations.

Be extremely neat, precise, and concise. It is important that you learn what to include and what to omit from your solutions. Staple your homework in the upper left corner, and begin each problem, in correct order, at the top of a new page (or side). (Sorry about the trees.)

All of the problems that will be assigned can be solved using material that we have discussed in class. Do not solve a problem by quoting a theorem in some reference, or by stating that the solution is an easy consequence of Theorem 5.5 in a book you found. All problems can and should be solved using the approaches and tools we have discussed in class.