Distributed Kalman Filtering for Sensor Networks

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- $w(k), v_i(k)$ are zero mean white Gaussian noise (WGN) with

$$E[w(k)w(l)^\top] = Q(k)\delta_{kl}$$

$$E[v_i(k)v_j(l)^\top] = R_i(k)\delta_{kl}\delta_{ij}$$

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- Define
  - estimate of the process state: $\hat{x}_k = E(x_k|Z_k), \bar{x}_k = E(x_k|Z_{k-1})$
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- Thus we want to perform KF for the system:
  - $x(k+1) = A_k x(k) + B_k w(k)$
  - $z(k) = H_k x(k) + v_k$
  - with $H_k = col(H_1(k), \ldots, H_n(k))$, $v_k = col(v_1(k), \ldots, v_n(k))$, $R_k = diag(R_1(k), \ldots, R_n(k))$

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Next: Perform distributed state estimation (or tracking) for the process

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Transformed update equations for the Central KF:

- $M_\mu(k) = (P_\mu^{-1}(k) + S(k))^{-1}$
- $\hat{x}(k) = \bar{x}(k) + M_\mu(k)(y(k) - S(k)\bar{x}(k))$
- $P_\mu(k+1) = A_k M_\mu(k) A_k^\top + B_k Q_\mu(k) B_k^\top$
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If each sensor implements a KF with above iterations, then all nodes have the same estimates as central estimate. Is it a DKF?

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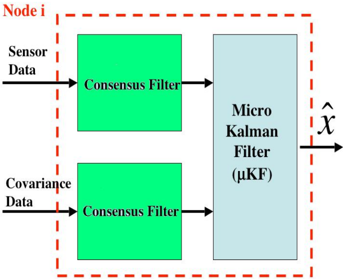

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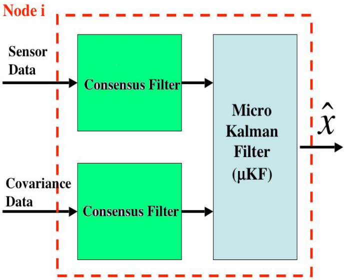
- Two consensus filters to compute $S(k)$ and $y(k)$ at each node using local information.



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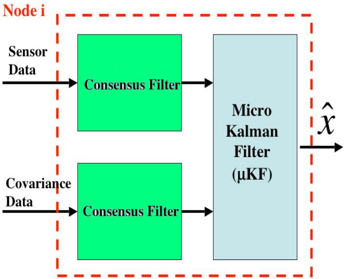

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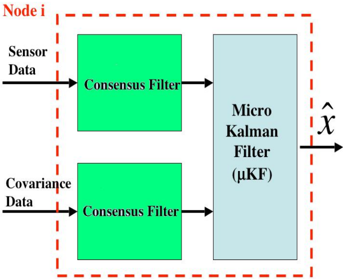
- Microfilters have identical structures.



Consensus Filters for DKF

- Use highpass consensus filters of the form
  - $\dot{q}_i = \beta \sum_{j \in N_i} (q_j - q_i) + \beta \sum_{j \in N_i} (u_j - u_i), \;\; \beta > 0$
  - $p_i = q_i + u_i$
  - where $\beta \sim O(1/\lambda_2)$ is relatively large.

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- u denotes the input for each node:
  - For CF1 [$\rightarrow y(k)$]: $u_j = H_j^\top R_j^{-1} z_j$
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- It is shown that for a connected network, outputs $p_i^1(k)$ and $p_i^2(k)$ of the highpass consensus filters asymptotically converge to $y(k)$ and $S(k)$.

DKF for Sensor Networks

- Node i sends the message: $msg_i = (q_i^1(k), q_i^2(k), u_i^1(k), u_i^2(k))$ to all of its neighbors. $\Rightarrow$ Message size is of dimension $O(m(m+1))$ with m being the dimension of the state x.

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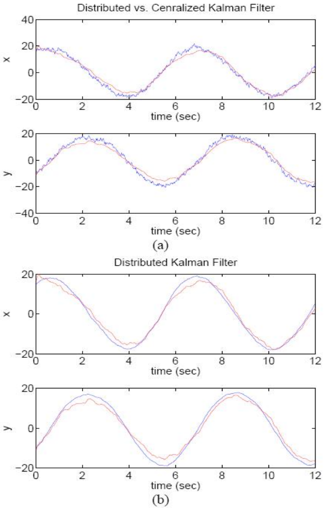
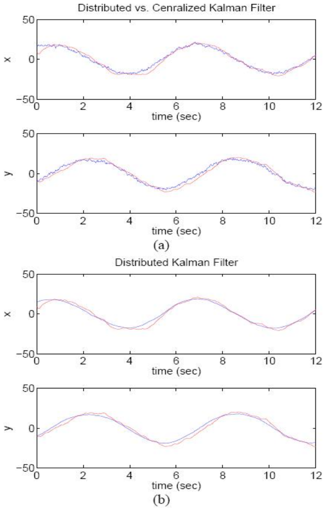
- The communication scheme is fully compatible with packet-based communication in real-world WSN.

DKF Results



Distributed position estimation for a moving object by node $i = 100$: (a) DKF vs. KF (DKF is the smooth curve in red) and (b) Distributed Kalman filter estimate (in red) vs. the actual position of the object (in blue).



Distributed position estimation for a moving object by node $i = 25$: (a) DKF vs. KF (DKF is the smooth curve in red) and (b) Distributed Kalman filter estimate (in red) vs. the actual position of the object (in blue).