3.26. Suppose you are to design a unity feedback controller for a first-order plant depicted in Fig. 3.50. (As you will learn in Chapter 4, the configuration shown is referred to as a proportional-integral controller.) You are to design the controller so that the closed-loop poles lie within the shaded regions shown in Fig. 3.51.

(a) What values of $\omega_n$ and $\zeta$ correspond to the shaded regions in Fig. 3.51? (A simple estimate from the figure is sufficient.)
(b) Let $K_p = \alpha = 2$. Find values for $K$ and $K_I$ so that the poles of the closed-loop system lie within the shaded regions.
(c) Prove that no matter what the values of $K$, $\alpha$ and $\omega$ are, the controller provides enough flexibility to place the poles anywhere in the complex (left-half) plane.

3.32. Consider the system shown in Fig. 3.55, where

$$G(s) = \frac{1}{s(s+3)} \quad \text{and} \quad D(s) = \frac{K(s+z)}{s+p}. \quad (3.82)$$

Find $K$, $z$, and $p$ so that the closed-loop system has a 10% overshoot to a step input and a settling time of 1.5 sec (1% criterion).
3.35. Consider the following second-order system with an extra pole:

\[ H(s) = \frac{\omega_n^2 p}{(s + p)(s^2 + 2\xi \omega_n s + \omega_n^2)}. \]

Show that the unit-step response is

\[ y(t) = 1 + A e^{-\xi t} + B e^{-\xi t} \sin(\omega_d t - \theta), \]

where

\[ A = \frac{-\omega_n^2}{\omega_n^2 - 2\xi \omega_n p + p^2}, \]
\[ B = \frac{p}{\sqrt{(p^2 - 2\xi \omega_n p + \omega_n^2)(1 - \xi^2)}}, \]
\[ \theta = \tan^{-1} \frac{\omega_n \sqrt{1 - \xi^2}}{p - \xi \omega_n}. \]

(a) Which term dominates \( y(t) \) as \( p \) gets large?
(b) Give approximate values for \( A \) and \( B \) for small values of \( p \).
(c) Which term dominates as \( p \) gets small? (Small with respect to what?)
(d) Using the preceding explicit expression for \( y(t) \) or the step command in MATLAB, and assuming that \( \omega_n = 1 \) and \( \xi = 0.7 \), plot the step response of the preceding system for several values of \( p \) ranging from very small to very large. At what point does the extra pole cease to have much effect on the system response?

Problem 4: Simulink
Refer to Problem 2 (Franklin 3.32). Simulate the system in 3.32 with the following parameters:

a. \( z = 3.1, p = 6.3, K = 20.25 \)
b. \( z = 5.77, p = 57.7, K = 222.45 \)
c. \( z = 3.1, K = 222.45, p = -2 \)

Plot the output \( y(t) \) for the above parameters and explain response in each case.

Problem 5: Modeling neural circuits
The amygdala is a region of the brain that is associated with emotions such as fear and anxiety. You have noted that in some cases of hereditary schizophrenia, patients express a mutated dopamine receptor in neurons that is refractory to excessive firing, i.e. the more frequently a neuron fires, the less sensitive the receptor is to dopamine, which reduces the rate of subsequent firing.

Let \( y(t) \) be the number of times a neuron fires. \( \dot{y}(t) \) is the firing rate of the neuron. \( \ddot{y}(t) \) is the rate of change in the firing rate, which is related to the level of dopamine stimulation \( u(t) \) in patients, as

\[ \ddot{y}(t) = -\dot{y}(t) + u(t). \]

a. Suppose \( y(0) = 0, \dot{y}(0) = 0 \), find the transfer function \( G(s) = \frac{Y(s)}{U(s)} \).

b. What is \( y(t) \) when there is a sudden increase in dopamine levels, i.e. \( u(t) = 1_+(t) \)?

c. A medical device company commissions you to design a controlled dopamine release device that stabilizes the amygdala in response to step changes in dopamine levels. Assume real time measurements of \( y(t) \) are available. The device releases dopamine according to \( u(t) = -e(t) + v(t) \), where \( v(t) \) is a user set signal and \( e(t) = K_1 y(t) + K_2 \dot{y}(t) \).
(a) Compute the transfer function, \( C(s) = \frac{E(s)}{Y(s)} \).

(b) Draw a block diagram showing the negative feedback circuit.

(c) Find the closed loop transfer function \( G_{CL}(s) = \frac{Y(s)}{V(s)} \).

(d) Find the values of \( K_1 \) and \( K_2 \) so that the closed loop transfer function \( G_{CL}(s) \) has poles at \( s = -2, s = -3 \).

d. In real life, instantaneous measurements of neural firing aren’t possible. Suppose you could only know what \( y(t) \) was after a delay \( \tau \), i.e. \( y(t - \tau) \).

(a) Assuming \( \tau \) is small, find the closed loop transfer function \( G_{CL}(s) \) in terms of \( \tau, K_1 \) and \( K_2 \).

(b) Suppose \( \tau = 0.1 \), find the values of \( K_1 \) and \( K_2 \) so that \( G_{CL}(s) \) continues to have poles at \( s = -2, s = -3 \).