# Image Primitives and Correspondence

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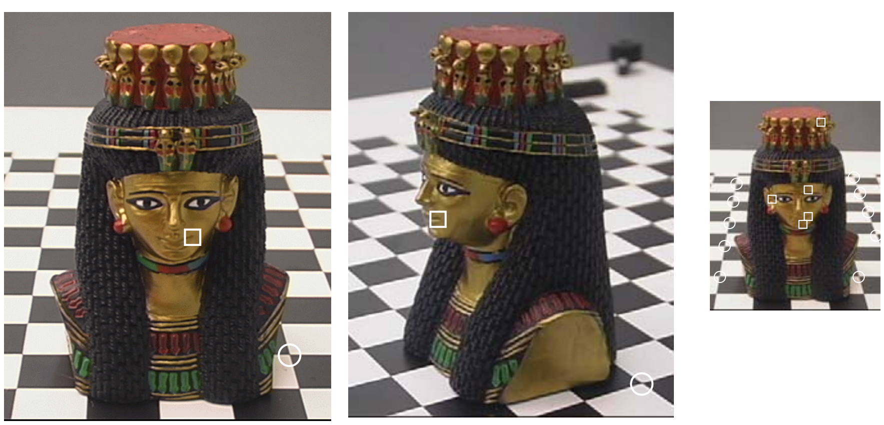
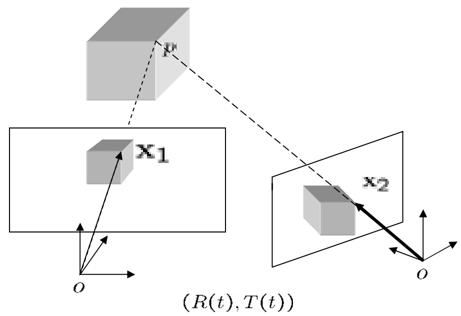
## Image Primitives and Correspondence



Given an image point in left image, what is the (corresponding) point in the right image, which is the projection of the same 3-D point

## Matching - Correspondence




Lambertian assumption $$I_1(\mathbf{x}_1) = \mathcal{R}(p) = I_2(\mathbf{x}_2)$$

Rigid body motion $$\mathbf{x}_2 = h(\mathbf{x}_1) = \frac{1}{\lambda_2(\mathbf{X})}(R\lambda_1(\mathbf{X})\mathbf{x}_1 + T)$$

Correspondence $$I_1(\mathbf{x}_1) = I_2(h(\mathbf{x}_1))$$



## Local Deformation Models

- Translational model

$$I_1(\mathbf{x}_1) = I_2(h(\mathbf{x}_1))$$

$$h(\mathbf{x}) = \mathbf{x} + d$$

- Affine model

$$I_1(\mathbf{x}_1) = I_2(h(\mathbf{x}_1))$$

$$h(\mathbf{x}) = A\mathbf{x} + d$$

- Transformation of the intensity values and occlusions

$$I_1(\mathbf{x}_1) = f_o(\mathbf{X}, g)I_2(h(\mathbf{x}_1) + n(h(\mathbf{x}_1))$$

## Feature Tracking and Optical Flow

- Translational model

$$I_1(\mathbf{x}_1) = I_2(\mathbf{x}_1 + \Delta\mathbf{x})$$

- Small baseline

$$I(\mathbf{x}(t), t) = I(\mathbf{x}(t) + \mathbf{u}dt, t + dt)$$

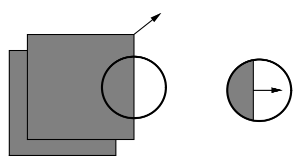
- RHS approx. by first two terms of Taylor series

$$\nabla I(\mathbf{x}(t), t)^T \mathbf{u} + I_t(\mathbf{x}(t), t) = 0$$

- Brightness constancy constraint



## Aperture Problem



- Normal flow

$$\mathbf{u}_n \doteq \frac{\nabla I^T \mathbf{u}}{||\nabla I||} \cdot \frac{\nabla I}{||\nabla I||} = -\frac{I_t}{||\nabla I||} \cdot \frac{\nabla I}{||\nabla I||}$$

## Optical Flow

- Integrate around over image patch

$$E_b(\mathbf{u}) = \sum_{W(x,y)} [\nabla I^T(x,y,t)\mathbf{u}(x,y) + I_t(x,y,t)]^2$$

- Solve

$$\begin{aligned} \nabla E_b(\mathbf{u}) &= 2 \sum_{W(x,y)} \nabla I(\nabla I^T \mathbf{u} + I_t) \\ &= 2 \sum_{W(x,y)} \left( \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \mathbf{u} + \begin{bmatrix} I_x I_t \\ I_y I_t \end{bmatrix} \right) \end{aligned}$$

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \mathbf{u} + \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix} = 0$$

$$G\mathbf{u} + \mathbf{b} = 0$$



## Optical Flow, Feature Tracking

$$\mathbf{u} = -G^{-1}\mathbf{b}$$

$$G = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

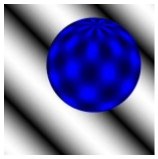
Conceptually:

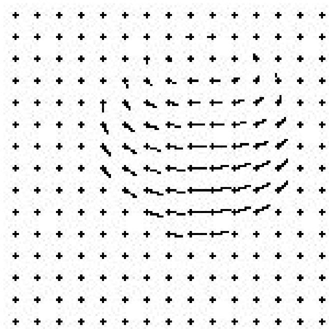
rank(G) = 0 blank wall problem
rank(G) = 1 aperture problem
rank(G) = 2 enough texture – good feature candidates

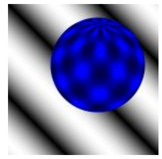
In reality: choice of threshold is involved

## Optical Flow

• Previous method - assumption locally constant flow





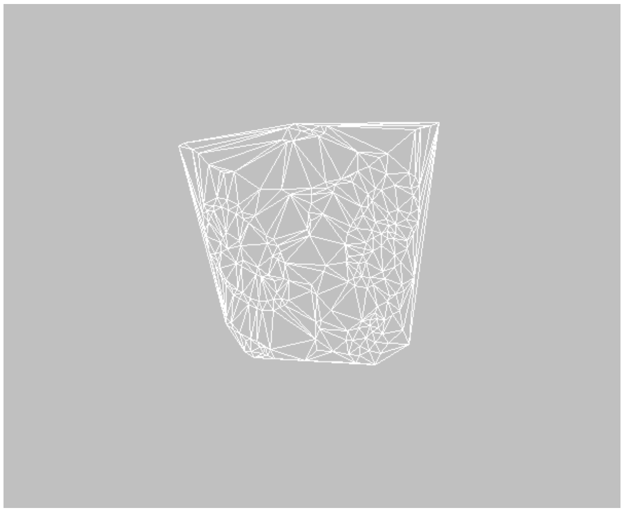


• Alternative regularization techniques (locally smooth flow fields, integration along contours)
• Qualitative properties of the motion fields



## Feature Tracking

## 3D Reconstruction - Preview





## Point Feature Extraction

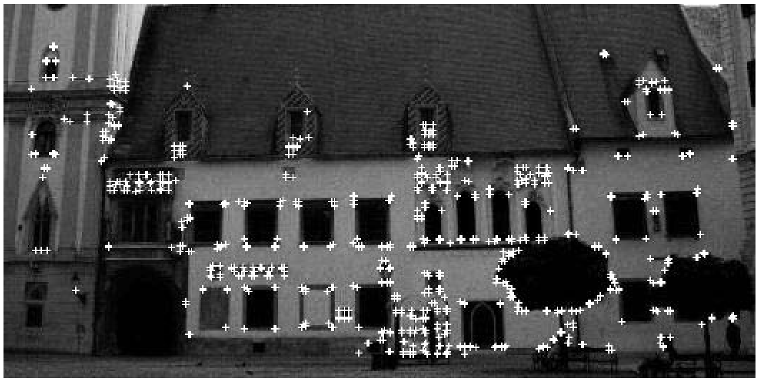
$$G = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

- Compute eigenvalues of G
- If smalest eigenvalue $\sigma$ of G is bigger than $\tau$ - mark pixel as candidate feature point

  - Alternatively feature quality function (Harris Corner Detector)

$$C(G) = \mathrm{det}(G) + k \cdot \mathrm{trace}^2(G)$$

## Harris Corner Detector - Example





## Wide Baseline Matching

## Region based Similarity Metric

- Sum of squared differences

$$SSD(h) = \sum_{\tilde{\mathbf{x}} \in W(\mathbf{x})} \|I_1(\tilde{\mathbf{x}}) - I_2(h(\tilde{\mathbf{x}})\|^2$$

- Normalize cross-correlation

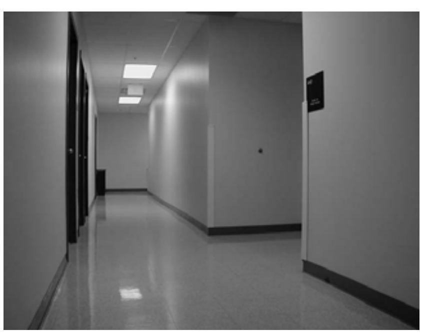
$$NCC(h) = \frac{\sum_{W(\mathbf{x})}(I_1(\tilde{\mathbf{x}})-\bar{I}_1)(I_2(h(\tilde{\mathbf{x}}))-\bar{I}_2))}{\sqrt{\sum_{W(\mathbf{x})}(I_1(\tilde{\mathbf{x}})-\bar{I}_1)^2 \sum_{W(\mathbf{x})}(I_2(h(\tilde{\mathbf{x}}))-\bar{I}_2)^2)}}$$

- Sum of absolute differences

$$SAD(h) = \sum_{\tilde{\mathbf{x}} \in W(\mathbf{x})} |I_1(\tilde{\mathbf{x}}) - I_2(h(\tilde{\mathbf{x}}))|$$



## Edge Detection



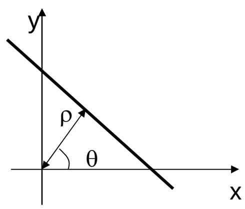
original image

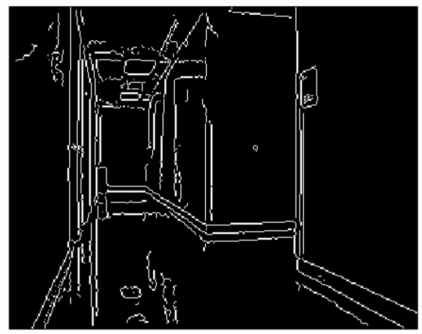
gradient magnitude

Canny edge detector

- Compute image derivatives
- if gradient magnitude > $\tau$ and the value is a local maximum along gradient direction – pixel is an edge candidate

## Line fitting



Non-max suppressed gradient magnitude

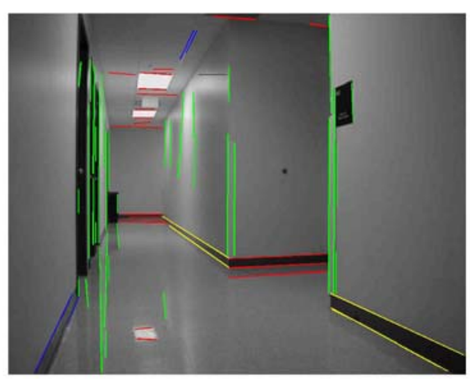
- Edge detection, non-maximum suppression
  (traditionally Hough Transform – issues of resolution, threshold selection and search for peaks in Hough space)
- Connected components on edge pixels with similar orientation
  - group pixels with common orientation



## Line Fitting

$$A = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i \\ \sum x_i y_i & \sum y_i^2 \end{bmatrix}$$

second moment matrix associated with each connected component



- Line fitting Lines determined from eigenvalues and eigenvectors of A
- Candidate line segments - associated line quality