HW 1: Advanced Topics in Computer Vision (580.464)

Instructor: René Vidal, Phone: 410-516-7306, E-mail: rvidal@cis.jhu.edu

Due 02/15/05 beginning of the class

1. Exercise 2.4 of MASKS.
2. Exercises 2.3 and 2.12 of MASKS. Do not use MATLAB or brute force in 2.12 part 1.
3. (a) Show that $\widehat{\omega}^2 = \omega\omega^T - \|\omega\|^2 I$ and $\widehat{\omega}^3 = -\|\omega\|^2\widehat{\omega}$.

   (b) Show that the Lie bracket between $\widehat{\omega_1}$ and $\widehat{\omega_2}$, $L = [\widehat{\omega_1}, \widehat{\omega_2}] = \widehat{\omega_1}\widehat{\omega_2} - \widehat{\omega_2}\widehat{\omega_1}$, is a skew-symmetric matrix, i.e., $L = -L^T = \hat{\omega}$ for some $\omega \in \mathbb{R}^3$. Find a formula for ω as a function of ω_1 and ω_2.

   (c) Show that $\exp(\widehat{\omega_1})\exp(\widehat{\omega_2}) = \exp(\widehat{\omega})$ for some $\omega \in \mathbb{R}^3$. Find a formula for ω as a function of ω_1 and ω_2.
4. (a) Implement a MATLAB function called `rodrigues.m` that takes as an input either a 3-vector or 3-vector and scalar or a 3×3 matrix and returns the corresponding rotation matrix or the 3-vector (or 3 vector and scalar) corresponding to the rotation axis. You should be able to call the function in one of the following ways: $R = \texttt{rodrigues}(\omega)$, $R = \texttt{rodrigues}(\omega, \theta)$, $\omega = \texttt{rodrigues}(R)$, $[\omega, \theta] = \texttt{rodrigues}(R)$. In case both ω and θ are input (or output) follow the convention of enforcing $\|\omega\| = 1$. You can check in MATLAB help how to use function with variable number of inputs and outputs by typing `help nargin, help nargout.`

   (b) Implement a MATLAB function called `skew.m` that takes as an input either a 3-vector or a 3×3 matrix and returns the corresponding skew-symmetric matrix or the 3-vector corresponding to skew-symmetric matrix.
5. Exercise 3.4 of MASKS.
6. Exercise 3.10 of MASKS.
7. (a) Derive the equations of the motion field $\mathbf{u} = f(\omega, v)$, induced by a camera moving with linear and angular velocity ω, v for the spherical projection model.

   (b) Derive the equations of the motion field of a planar surface $\mathbf{u} = f(\omega, v, \pi)$, where P is a 3-D plane $N^T\mathbf{X} = d$ with normal $N = [a, b, c]^T$ observed by a camera moving with linear and angular velocity ω, v. How well does the affine flow model approximates the motion field of a plane moving in 3D ?
8. (a) Implement a MATLAB function `x = project(X,type)` that takes a matrix `X` $\in \mathbb{R}^{3\times P}$ whose columns are points in $\mathbb{R}^3$ and a type of projection `type` (orthographic, perspective, spherical or paracatadioptric) and returns the projection of these points `x` $\in \mathbb{R}^{3\times P}$ onto the image retina according to the given model. Your implementation should contain no `for` loops.

   (b) Implement a MATLAB function `u = optflow(w,v,X,type)` that takes a rotational velocity `w`, a translational velocity `v`, a matrix `X` $\in \mathbb{R}^{3\times P}$ whose columns are points in $\mathbb{R}^3$ and a type of projection `type` (orthographic, perspective, spherical or paracatadioptric) and returns the optical flow of these points `u` $\in \mathbb{R}^{2\times P}$ according to the given projection model. Your implementation should contain no `for` loops.