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Multi-Scale Hybrid Linear Models for Lossy Image Representation

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MULTI-SCALE IMPLEMENTATION

- IMAGE SPACE DOMAIN
- WAVELET DOMAIN

OTHER APPLICATIONS

CONCLUSIONS AND FUTURE DIRECTIONS

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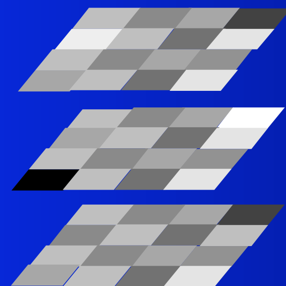
Introduction – Image Representation via Linear Transformations



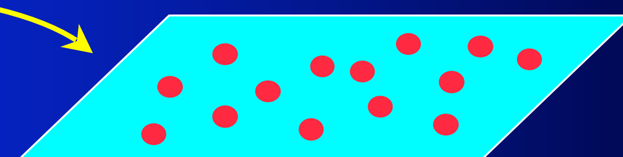
better representations?

pixel-based representation

three matrixes of RGB-values

linear transformation

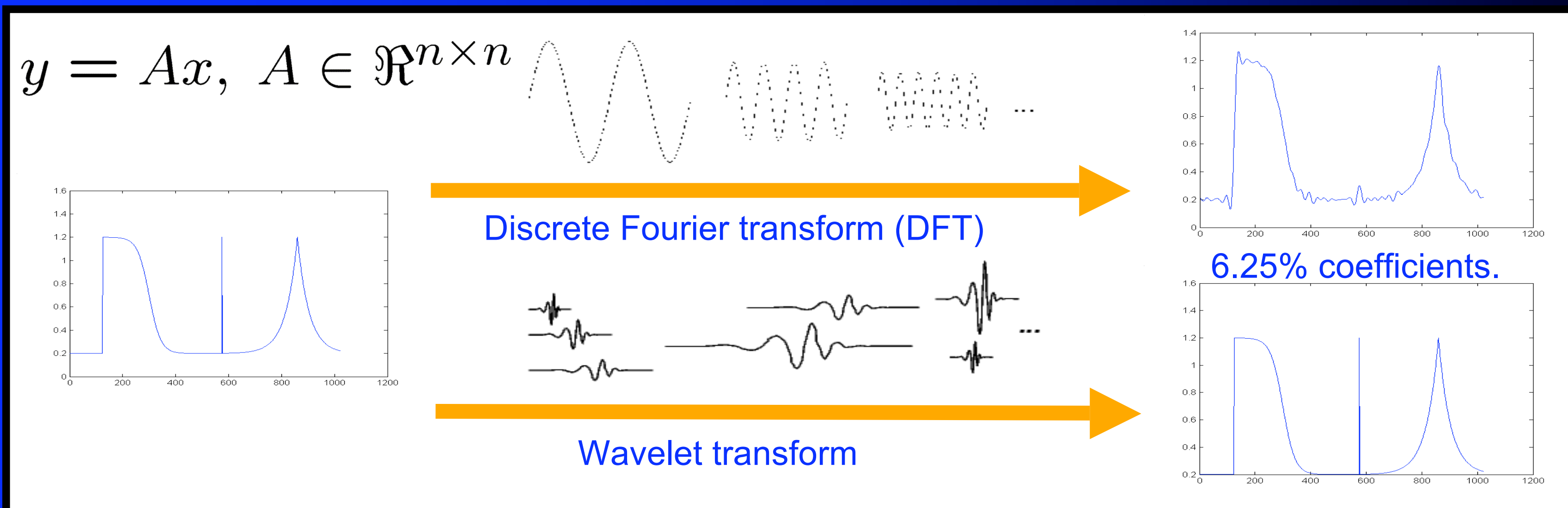
a more compact representation



Introduction

Fixed Orthogonal Bases (representation, approximation, compression)

- Discrete Fourier transform (DFT) or discrete cosine transform (DCT) (Ahmed '74): JPEG.
- Wavelets (multi-resolution) (Daubechies'88, Mallat'92): JPEG-2000.
- Curvelets and contourlets (Candes & Donoho'99, Do & Veterlli'00)



Unorthogonal Bases (for redundant representations)

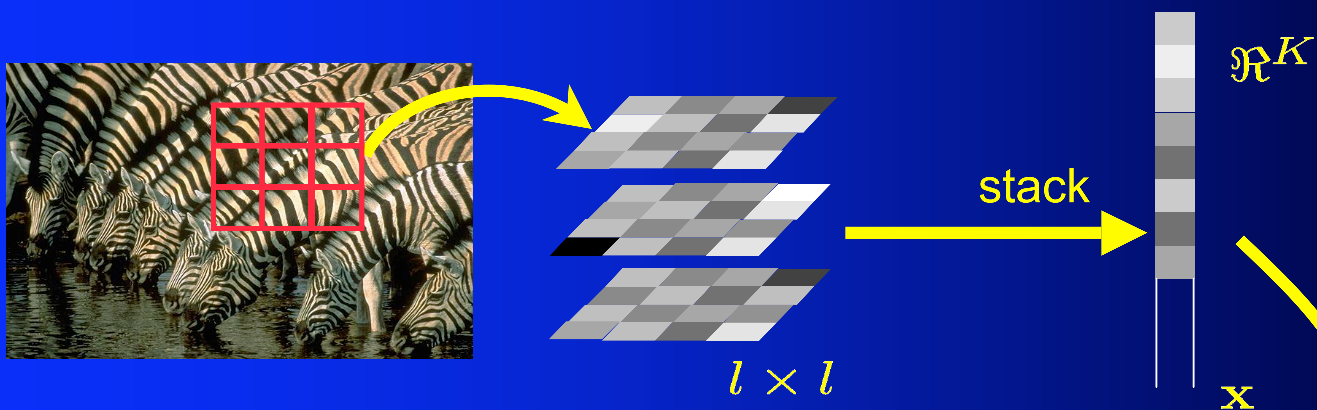
- Extended lapped transforms, frames, sparse representations (L^p geometry)…

$$\min |x|, \text{ s.t } y = Ax, \; A \in \Re^{n \times m}, m > n$$

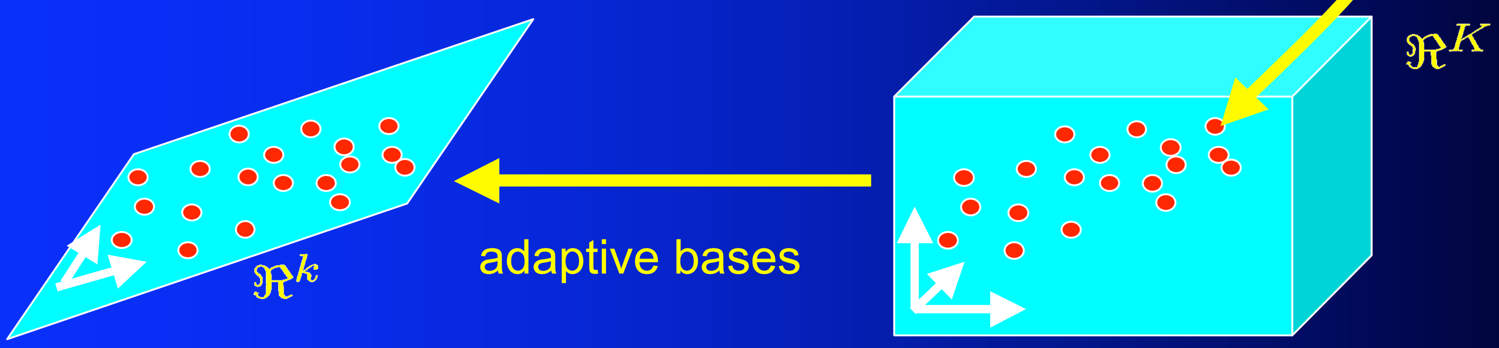
Introduction

Adaptive Bases (optimal if imagery data are uni-modal)

- Karhunen-Loeve transform (KLT), also known as PCA (Pearson’1901, Hotelling’33, Jolliffe’86)



$$y = Ax, \; A \in \Re^{m \times n}, m < n$$

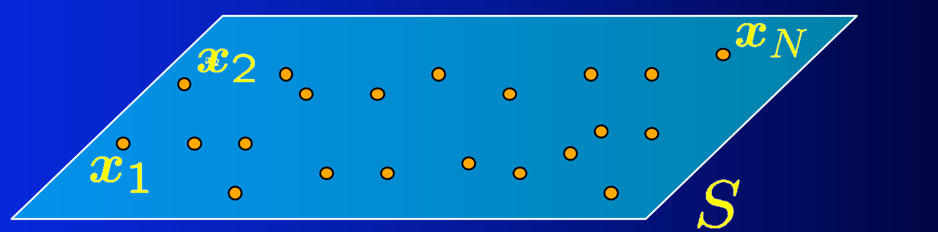


Introduction – Principal Component Analysis (PCA)

Dimensionality Reduction

Find a low-dimensional representation (model) for high-dimensional data.

Principal Component Analysis (Pearson'1901, Hotelling'1933, Eckart & Young'1936) or Karhunen-Loeve transform (KLT).



$$U \Sigma V^T = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots \boldsymbol{x}_N] \in \Re^{K \times N}$$

Basis for S

SVD

Variations of PCA

- Nonlinear Kernel PCA (Scholkopf-Smola-Muller'98)
- Probabilistic PCA (Tipping-Bishop'99, Collins et.al'01)
- Higher-Order SVD (HOSVD) (Tucker'66, Davis'02)
- Independent Component Analysis (Hyvarinen-Karhunen-Oja'01)

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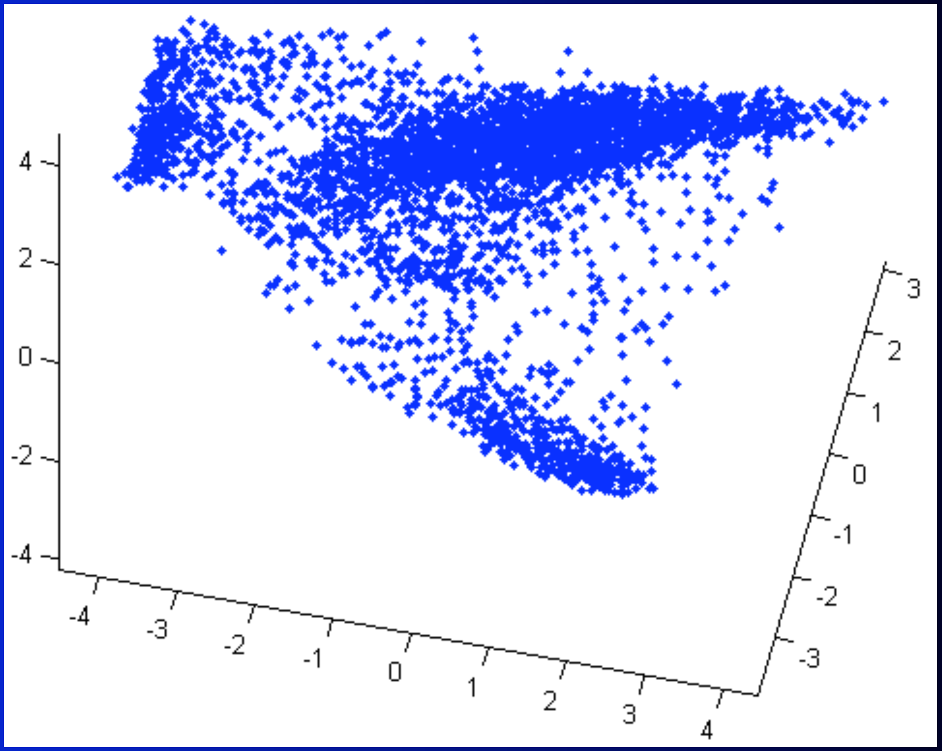
Hybrid Linear Models – Multi-Modal Characteristics

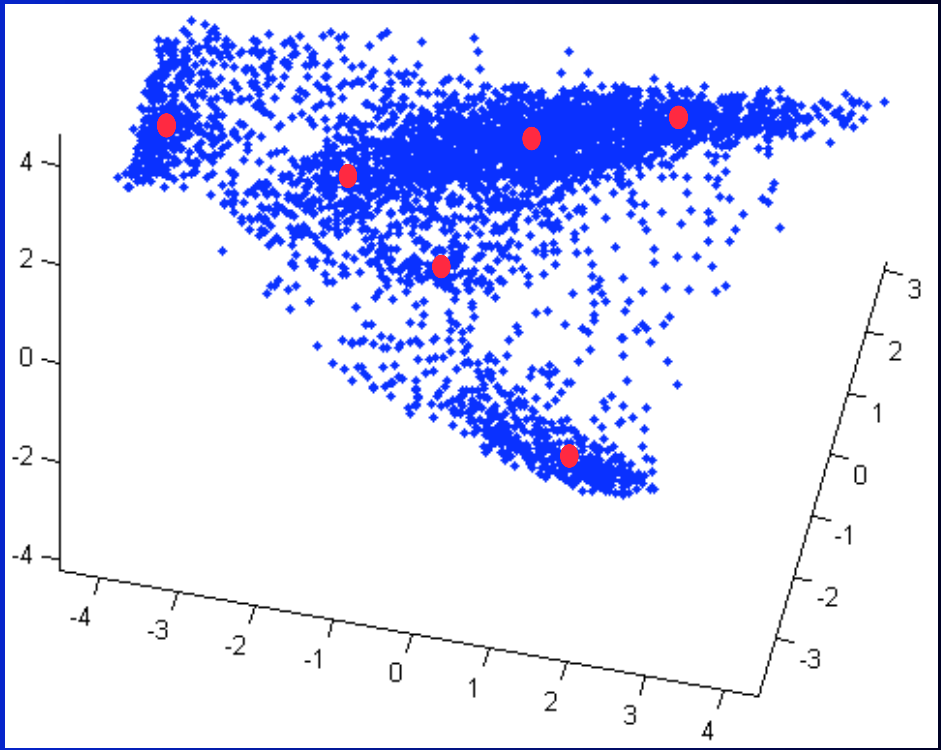
Distribution of the first three principal components of the Baboon image: A clear multi-modal distribution



Hybrid Linear Models – Multi-Modal Characteristics

Vector Quantization (VQ)

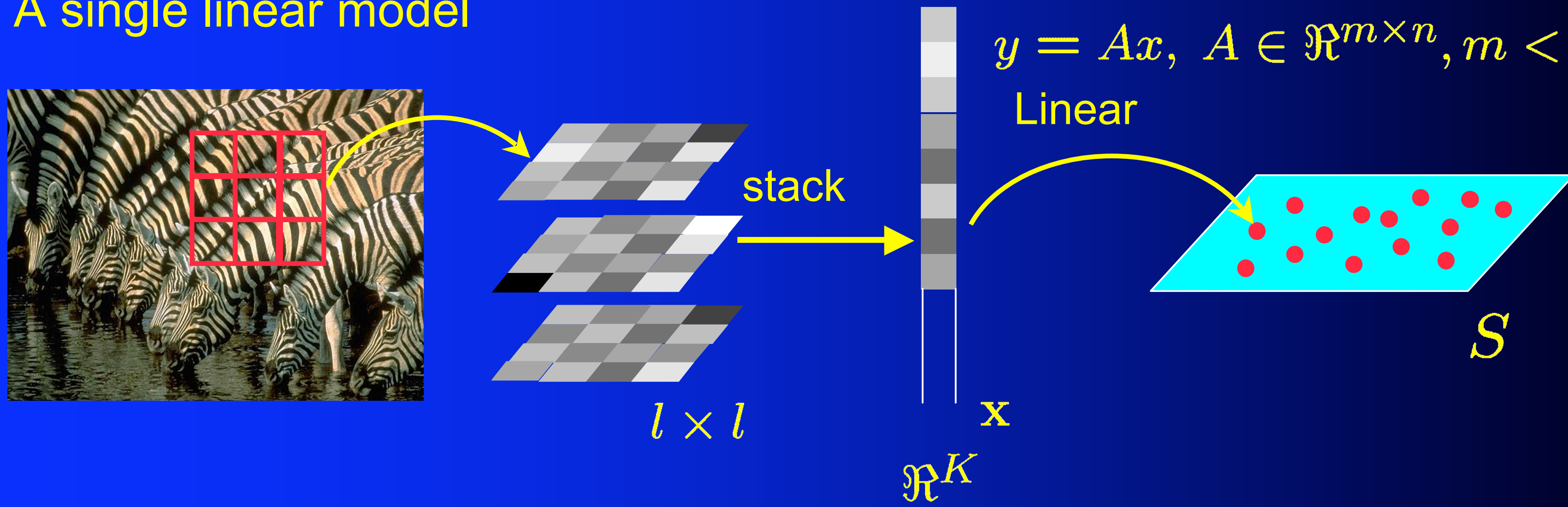
- multiple 0-dimensional affine subspaces (i.e. cluster means)
- existing clustering algorithms are iterative (EM, K-means)



Hybrid Linear Models – Versus Linear Models

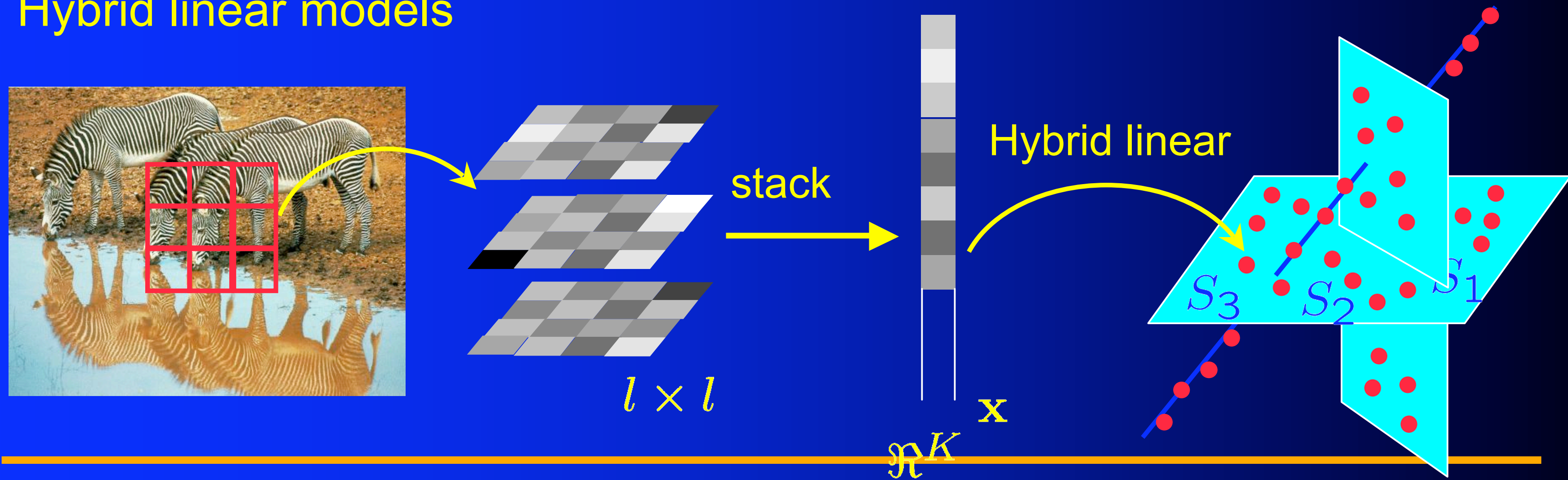
A single linear model

$$y = Ax,\ \ A \in \Re^{m\times n}, m < n$$



Hybrid linear models

$$y = A_i x,\ \ A_i \in \Re^{m_i\times n}, m_i < m < n$$



Hybrid Linear Models – Characteristics of Natural Images

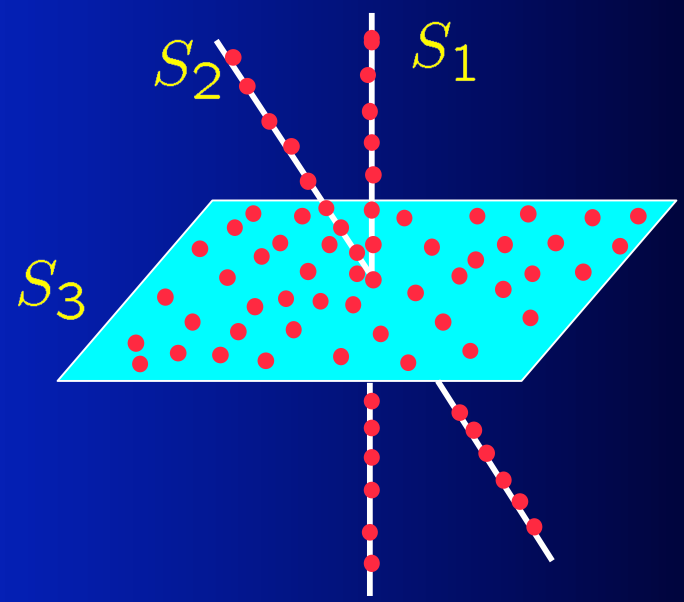
	Multivariate 1D	Multivariate 2D	Hybrid (multi-modal)	Hierarchical (multi-scale)	High-dimension (vector-valued)
Fourier (DCT)	X	X			
Wavelets	X			X	
Curvelets		X			
Random fields		X	X	X	
PCA/KLT	X	X			X
VQ	X	X	X		X
Hybrid linear	X	X	X	X	X

We need a new & simple paradigm to effectively account for all these characteristics simultaneously.

Hybrid Linear Models – Subspace Estimation and Segmentation

Hybrid Linear Models (or Subspace Arrangements)

- the number of subspaces is unknown
- the dimensions of the subspaces are unknown
- the basis of the subspaces are unknown
- the segmentation of the data points is unknown



"Chicken-and-Egg" Coupling

- Given segmentation, estimate subspaces
- Given subspaces, segment the data

Hybrid Linear Models – Some Related Literature

- **Heuristic Approaches** (Boult et.al, Costeira et.al, Kanatani,…)
  - Segment data using similarity matrices + clustering
  - Eigenvector (spectral) segmentation (…, Vempala-Wang'02)

- **Iterative Approaches**
  - Generative model: data membership + mixture model
  - Identify subspaces using Expectation Maximization
    - E-step: estimate membership given model parameters
    - M-step: estimate model parameters given membership
  - Probabilistic PCA (Tipping-Bishop'99), K-subspaces (Ho et. al'03), subspace growing and selection (Leonardis et. al'02)

Is there a non-iterative solution to the subspace estimation & segmentation problem?

Hybrid Linear Models – Generalized Principal Component Analysis

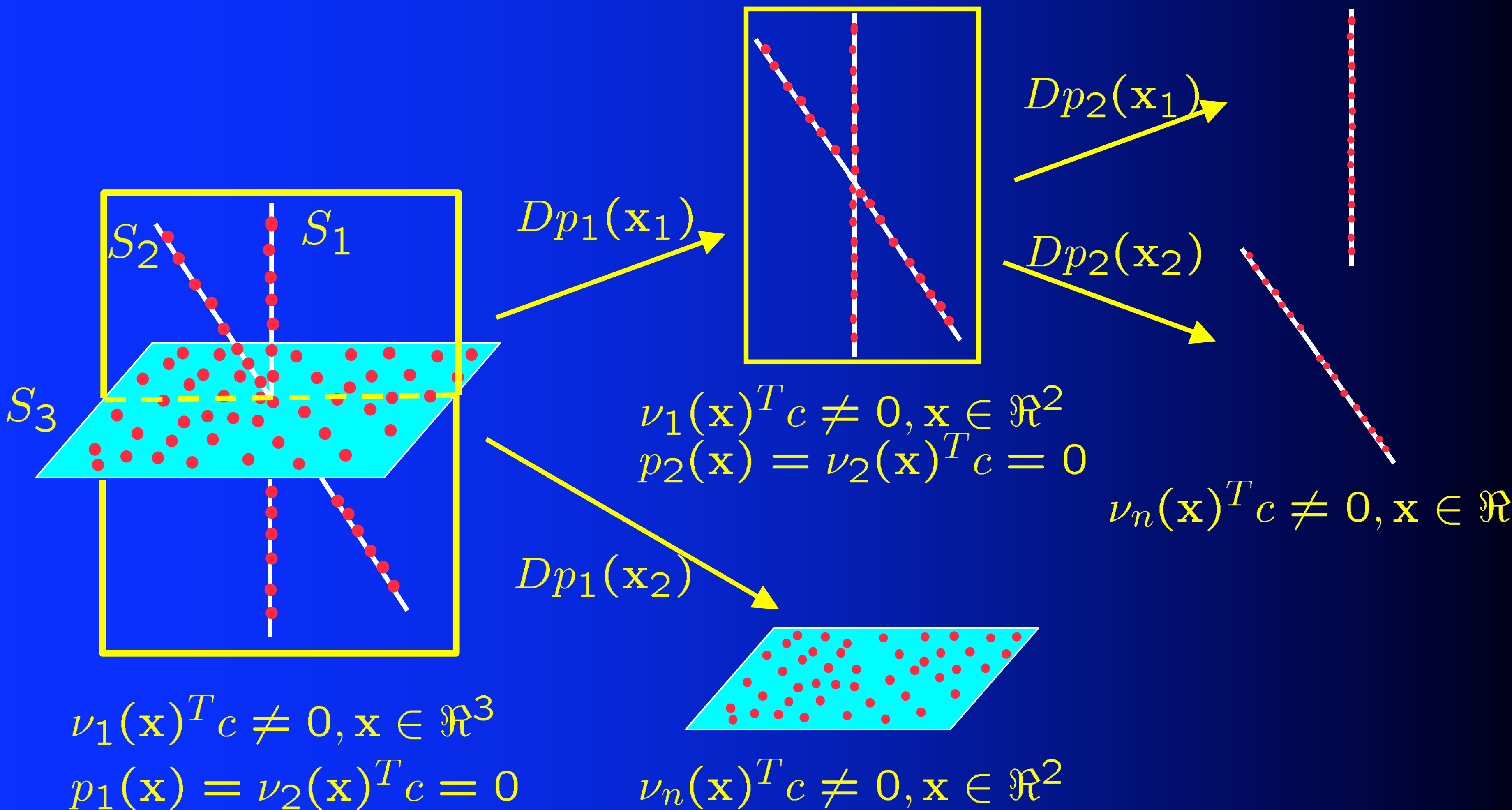
- **Generalized PCA** (Vidal-Ma-Sastry'03,04) – Sketch:
  - Fit all data points with a set of polynomials of the lowest degree
  - Select one representative point on each subspace
  - Derivatives of the polynomials at the point are normal vectors to the subspace
  - Segment the data points into different subspaces

In the absence of noise,

- the solution is closed-form (no initialization), and
- the algorithm uses only linear algebraic techniques.

Hybrid Linear Models – Recursive Version (an Example)

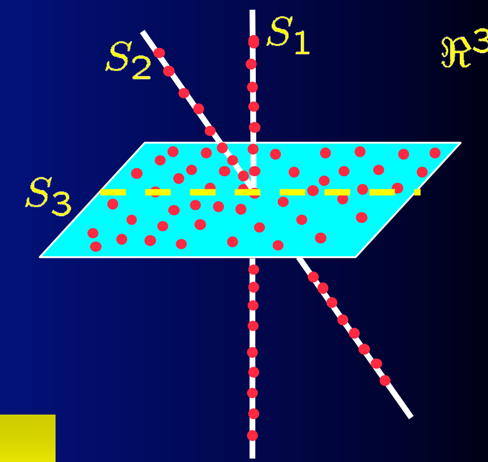
$$p(\mathbf{x}) = c_1 x_1^n + c_2 x_1^{n-1} x_2 + \cdots + c_m x_3^n = \nu_n(\mathbf{x})^T c$$



Hybrid Linear Models – Effective Dimension

Model Selection (for Noisy Data)

- Model complexity;
- Data fidelity;



Number of subspaces

$$ED(\mathbf{X}, S) \doteq \frac{1}{N}\sum_{1}^{s} k_i(K - k_i) + \frac{1}{N}\sum_{1}^{s} N_i k_i$$

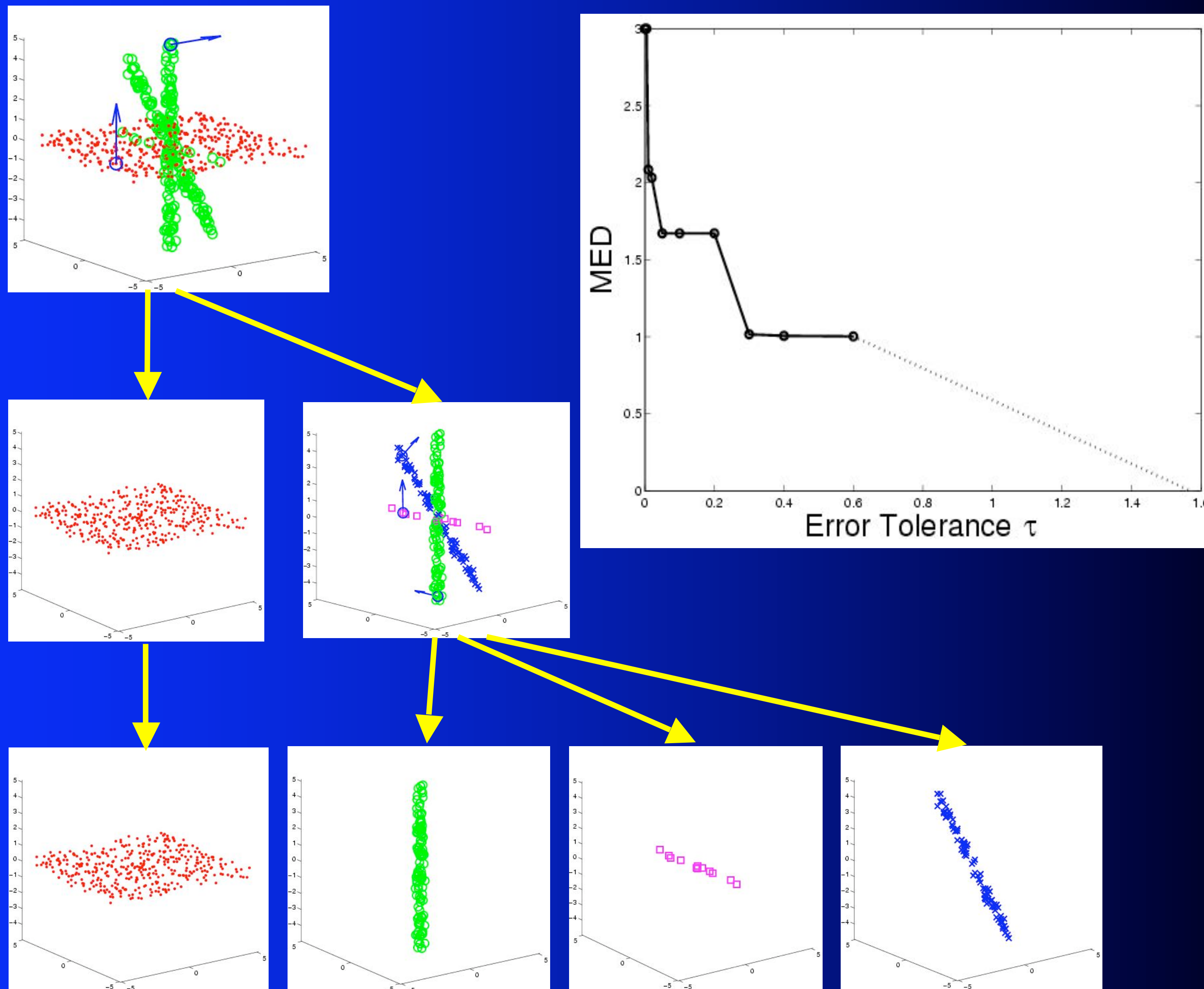
Total number of points

Dimension of each subspace

Number of points in each subspace

Model selection criterion: minimizing effective dimension subject to a given error tolerance (or PSNR)

Hybrid Linear Models – Simulation Results (5% Noise)

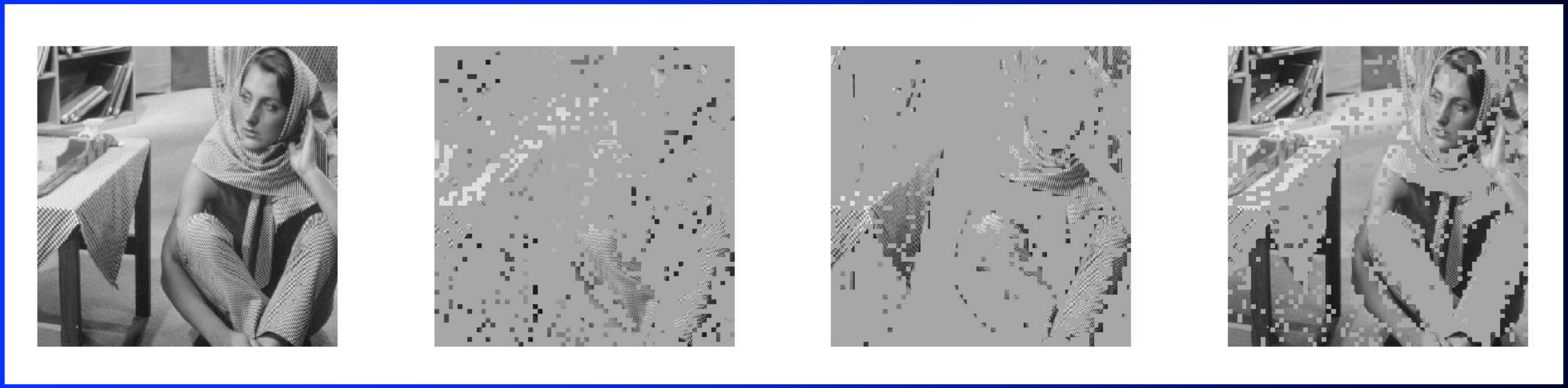


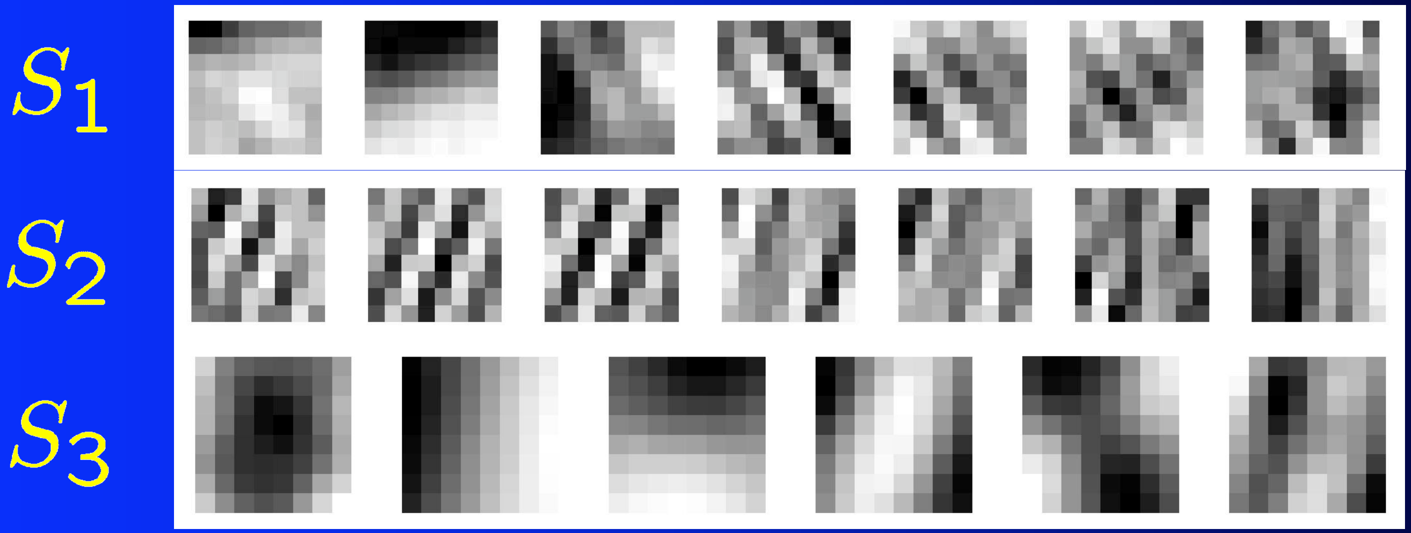
ED=3

ED=2.0067

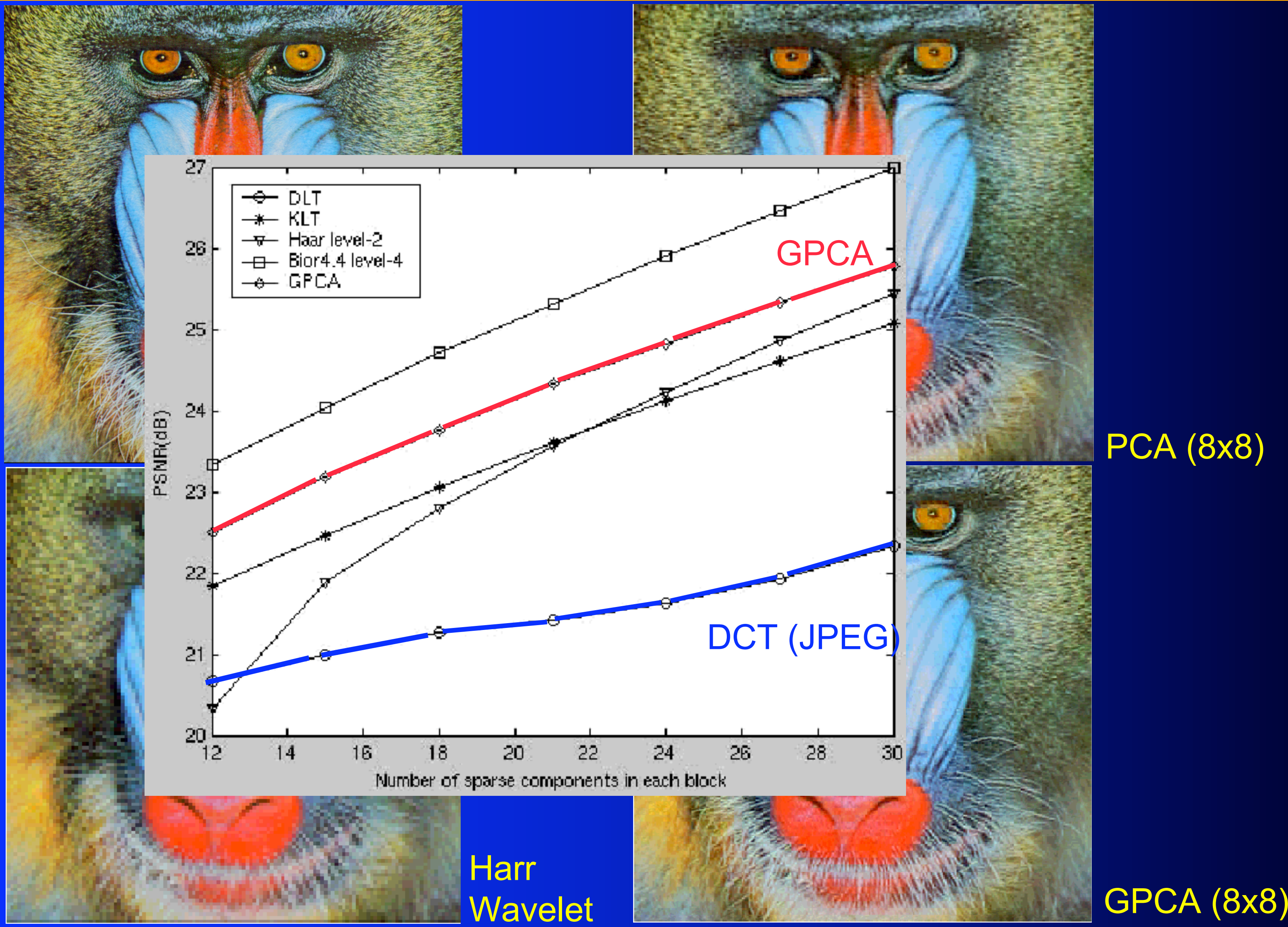
ED=1.6717

Hybrid Linear Models – Subspaces of the Barbara Image





Hybrid Linear Models – Lossy Image Representation (Baboon)



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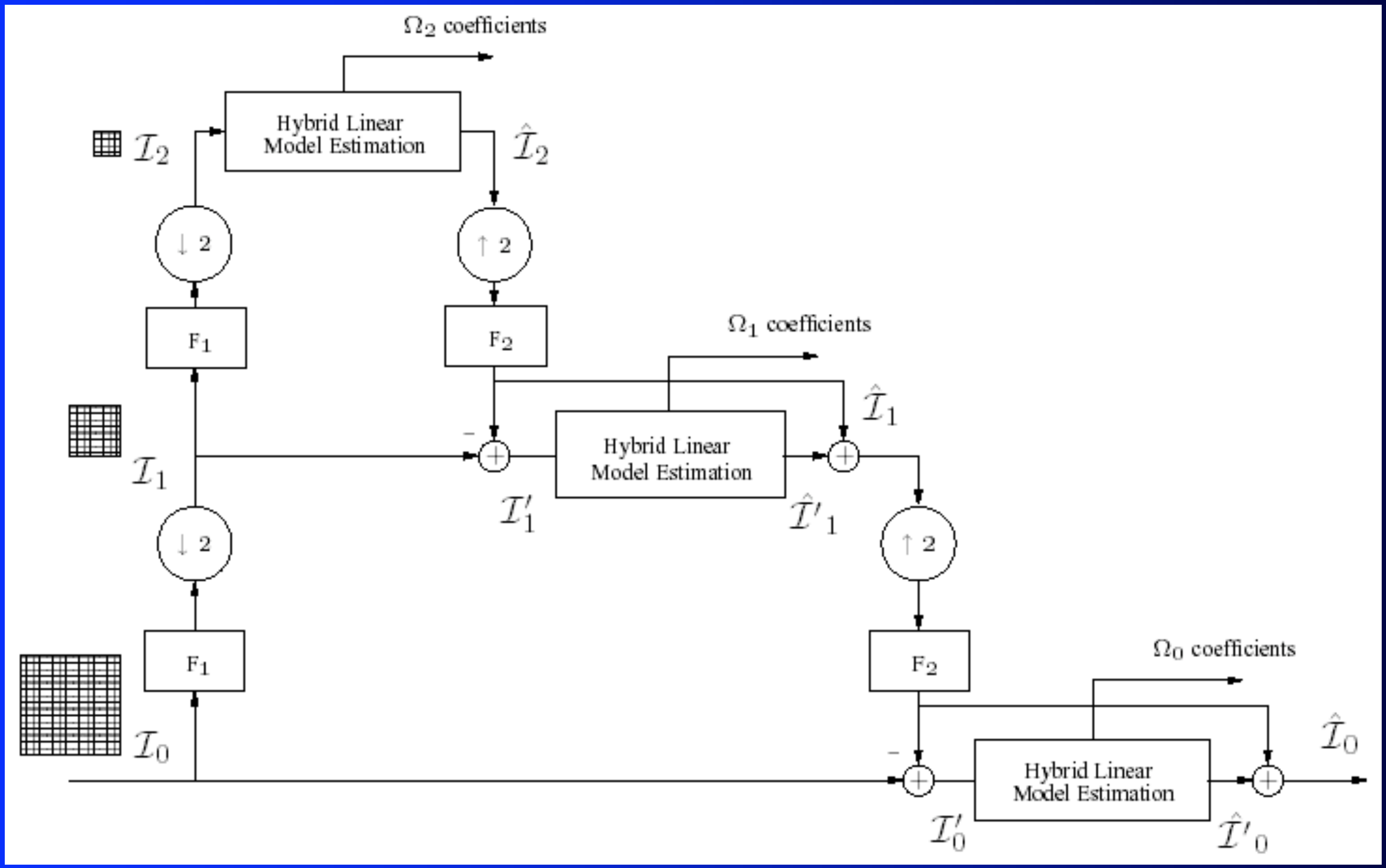
Multi-Scale Implementation – Algorithm Diagram

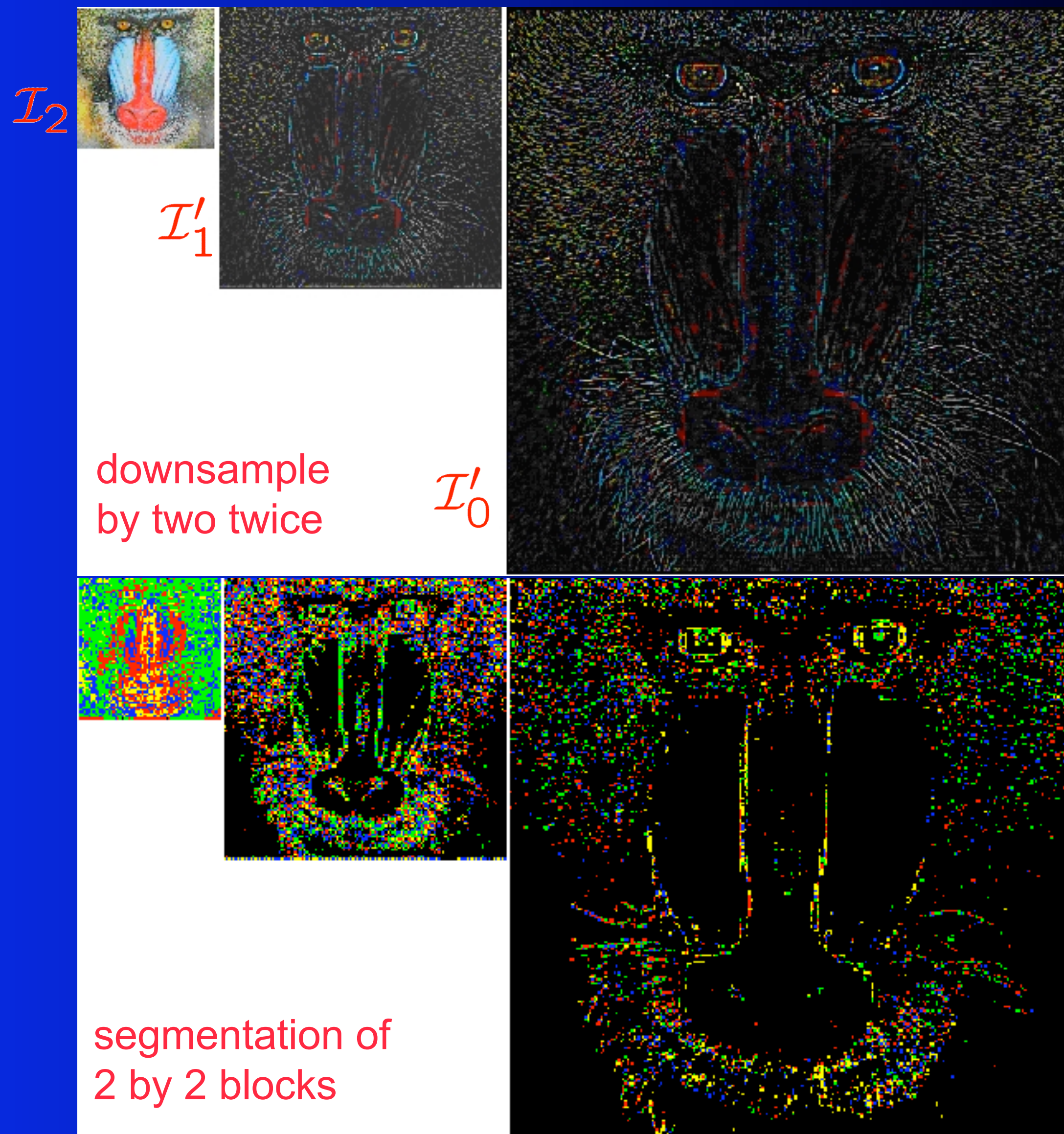
Diagram for a level-3 implementation of hybrid linear models for image representation



Multi-Scale Implementation – The Baboon Image

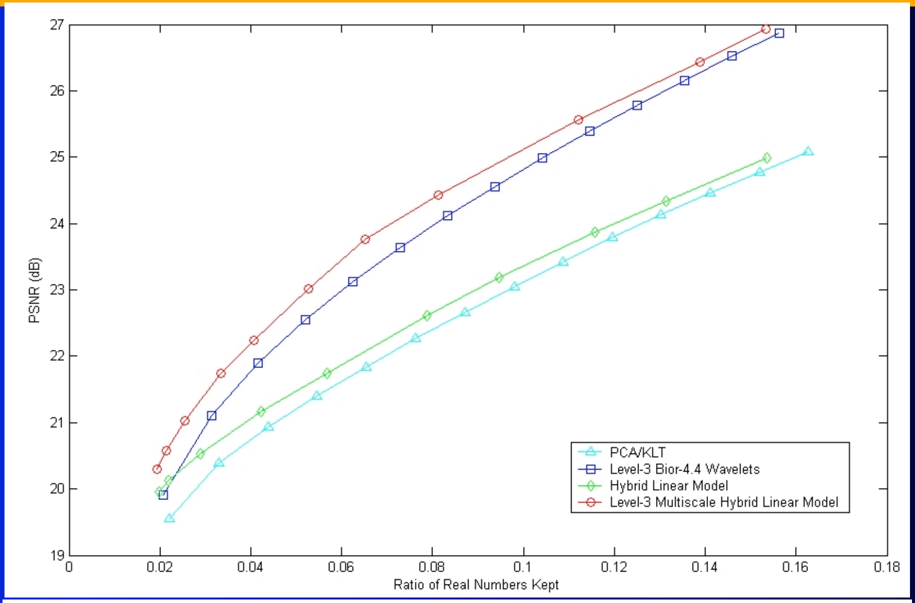
The Baboon image

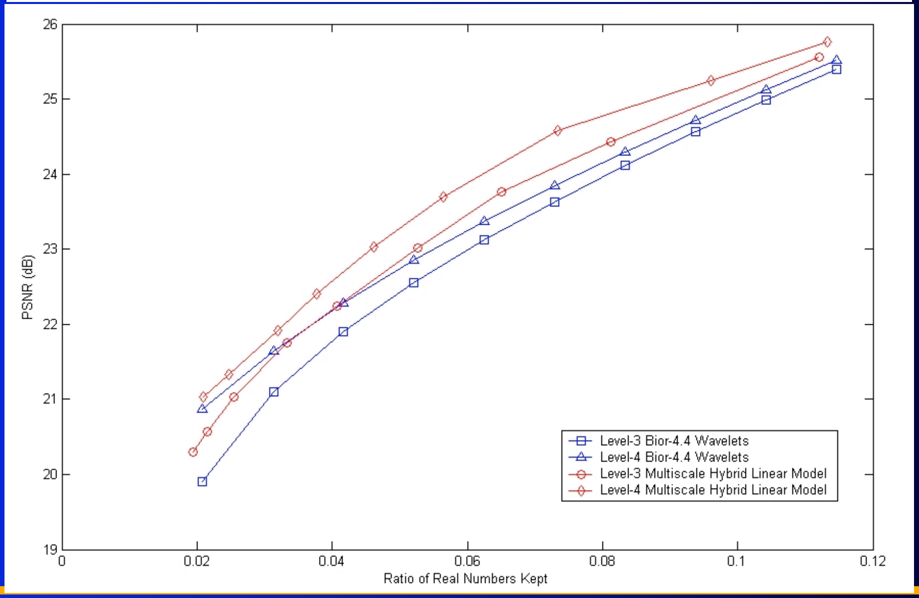
$\mathcal{I}$



Multi-Scale Implementation – Comparison with Other Methods

The Baboon image

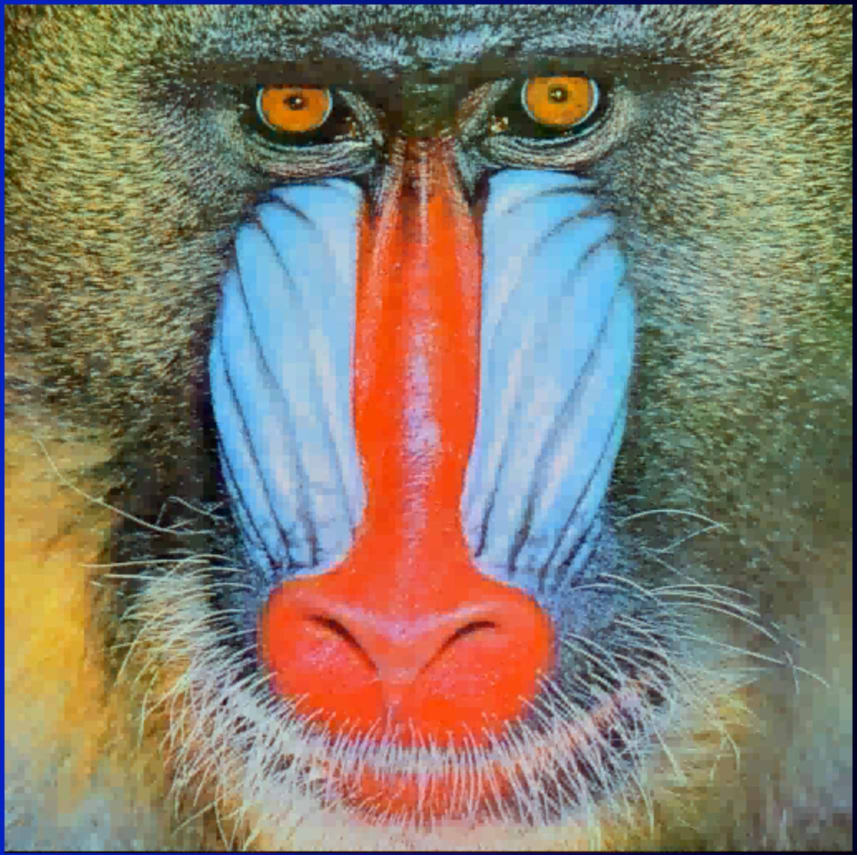




Multi-Scale Implementation – Image Approximation

Comparison with level-3 wavelet (7.5% coefficients)

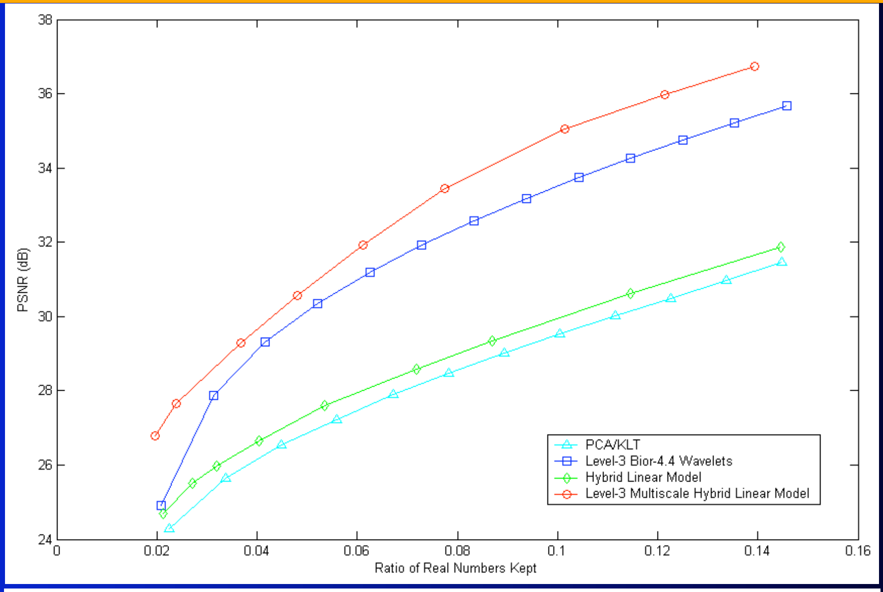
Level-3 bior-4.4 wavelets
PSNR=23.94

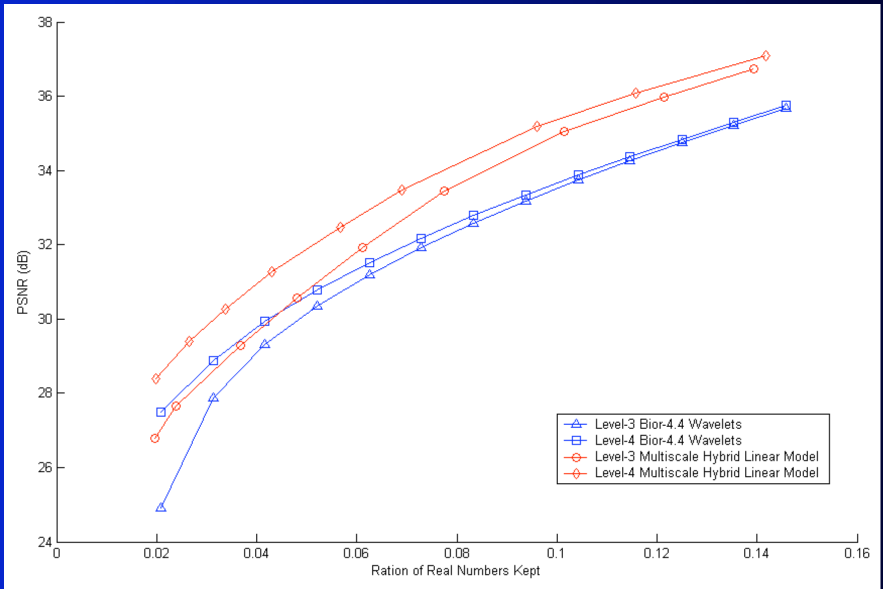


Level-3 hybrid linear model
PSNR=24.64

Multi-Scale Implementation – More Comparison

The Hill image

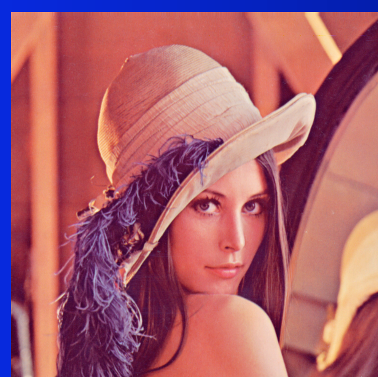




Multi-Scale Implementation – Other Test Images

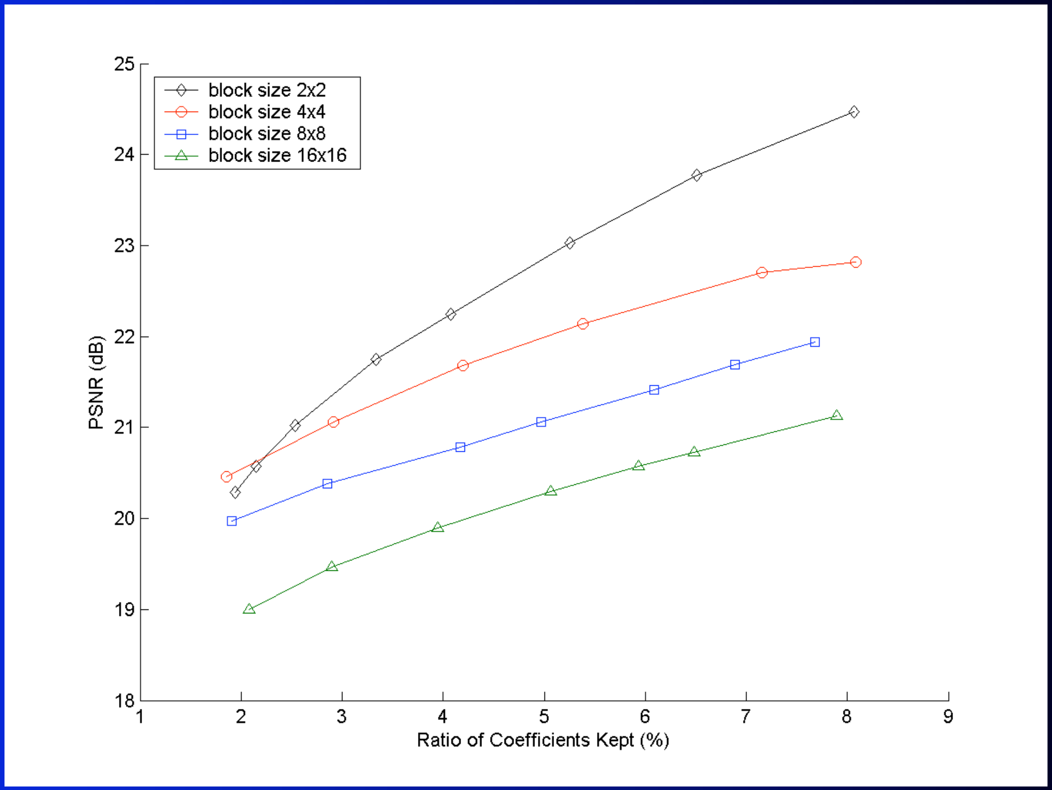
Outperform wavelets except for the two:

Reexamine later in wavelet domain



Multi-Scale Implementation – Block Size Effect

The Baboon image



Some problems with the multi-scale hybrid linear model:

1. has minor block effect;
2. is computationally more costly (than Fourier, wavelets, PCA);
3. does not fully exploit spatial smoothness as wavelets.

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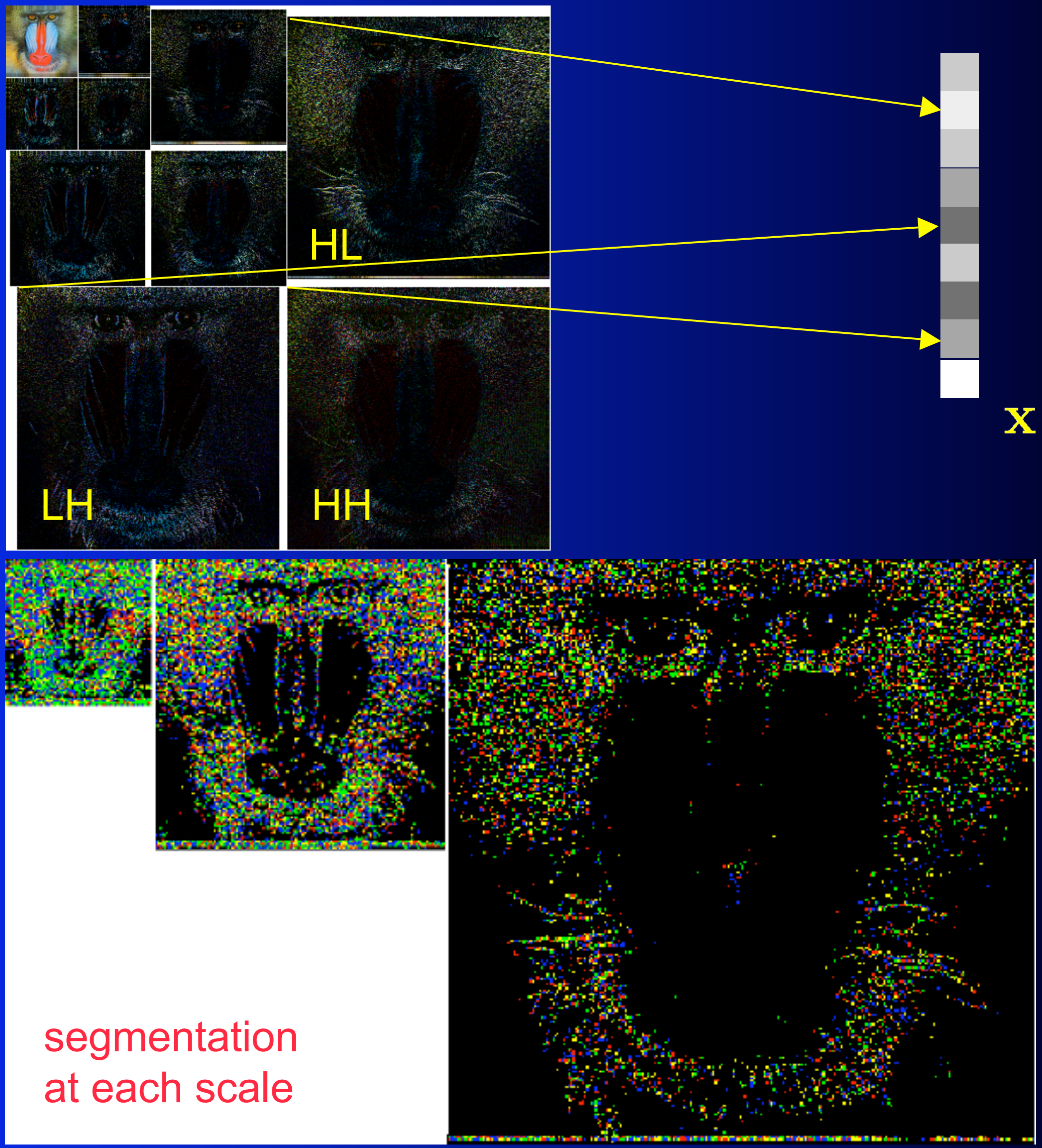
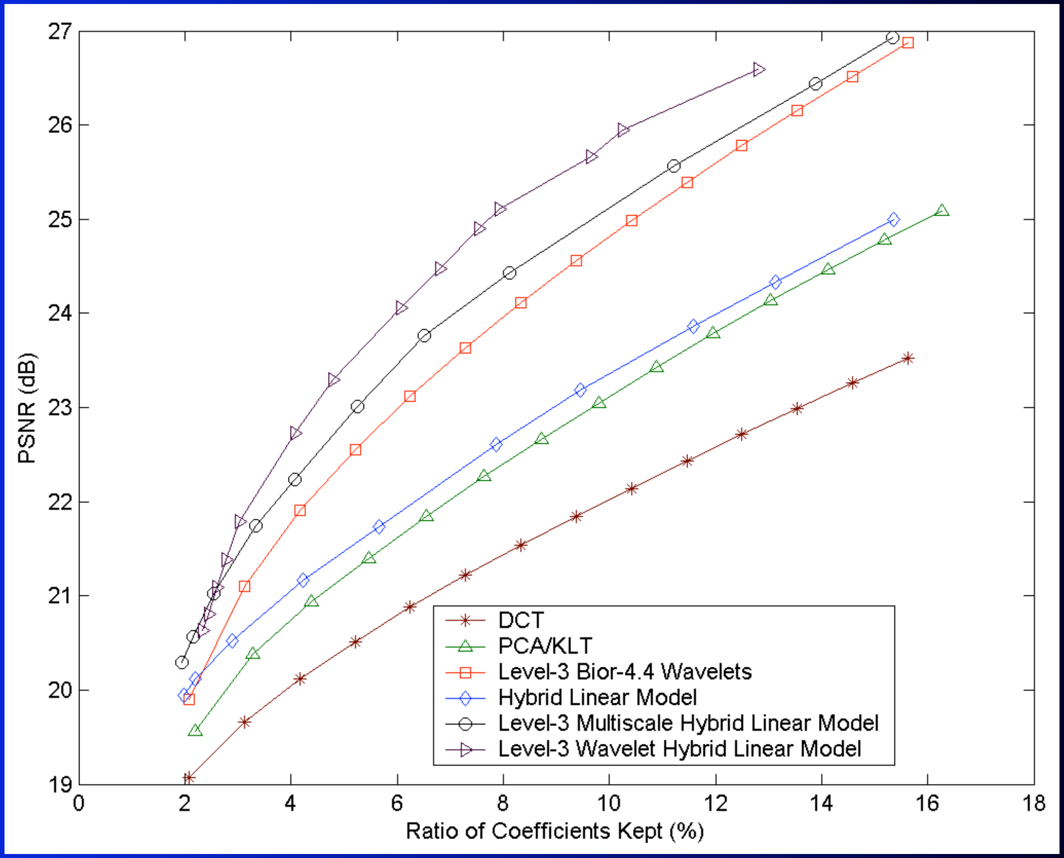
Multi-Scale Implementation – The Wavelet Domain

The Baboon image



Multi-Scale Implementation – Wavelets v.s. Hybrid Linear Wavelets

The Baboon image



Advantages of the hybrid linear model in wavelet domain:

1. eliminates block effect;
2. is computationally less costly (than in the spatial domain);
3. achieves higher PSNR.

Multi-Scale Implementation – Visual Comparison

Comparison among several models (7.5% coefficients)

Original Image

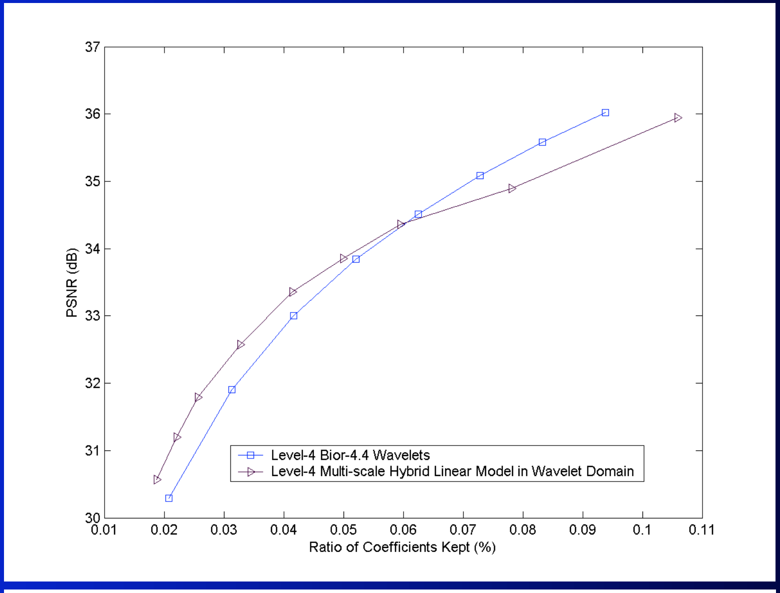
Wavelets
PSNR=23.94

Hybrid model in spatial domain
PSNR=24.64

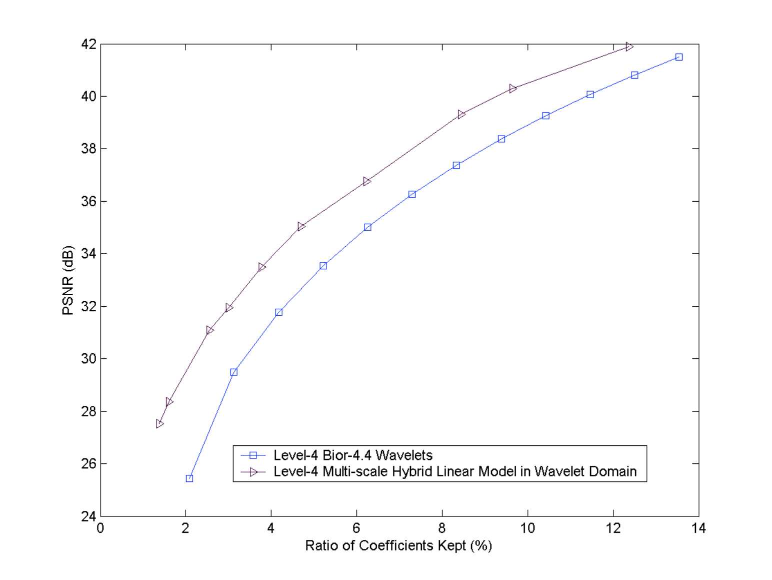
Hybrid model in wavelet domain
PSNR=24.88

Multi-Scale Implementation – Wavelets v.s. Hybrid Linear Wavelets

The Lena image



The Monarch image



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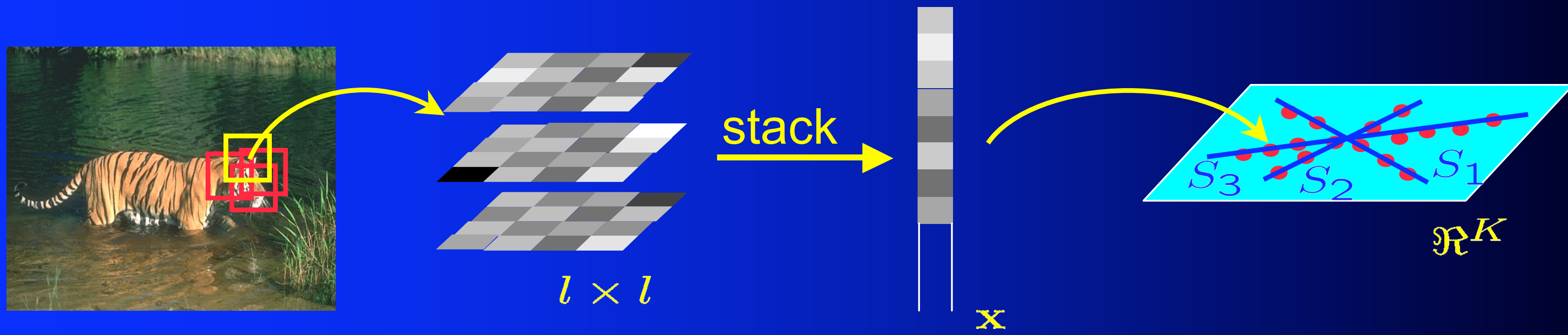
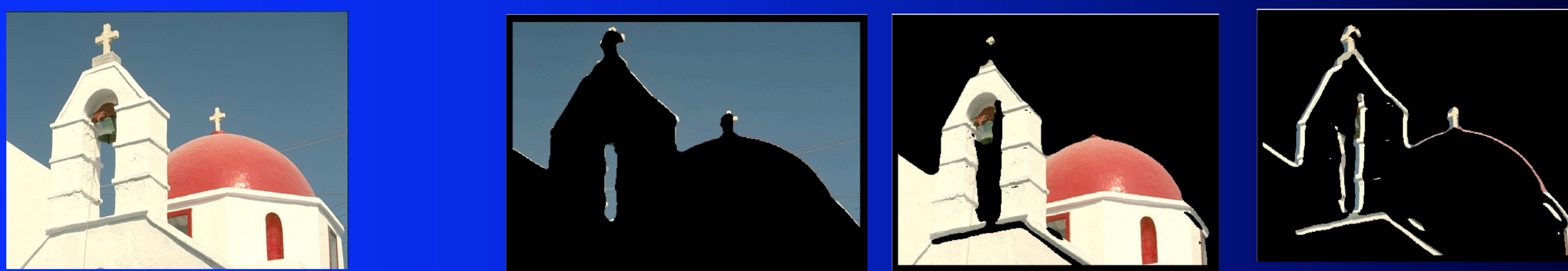
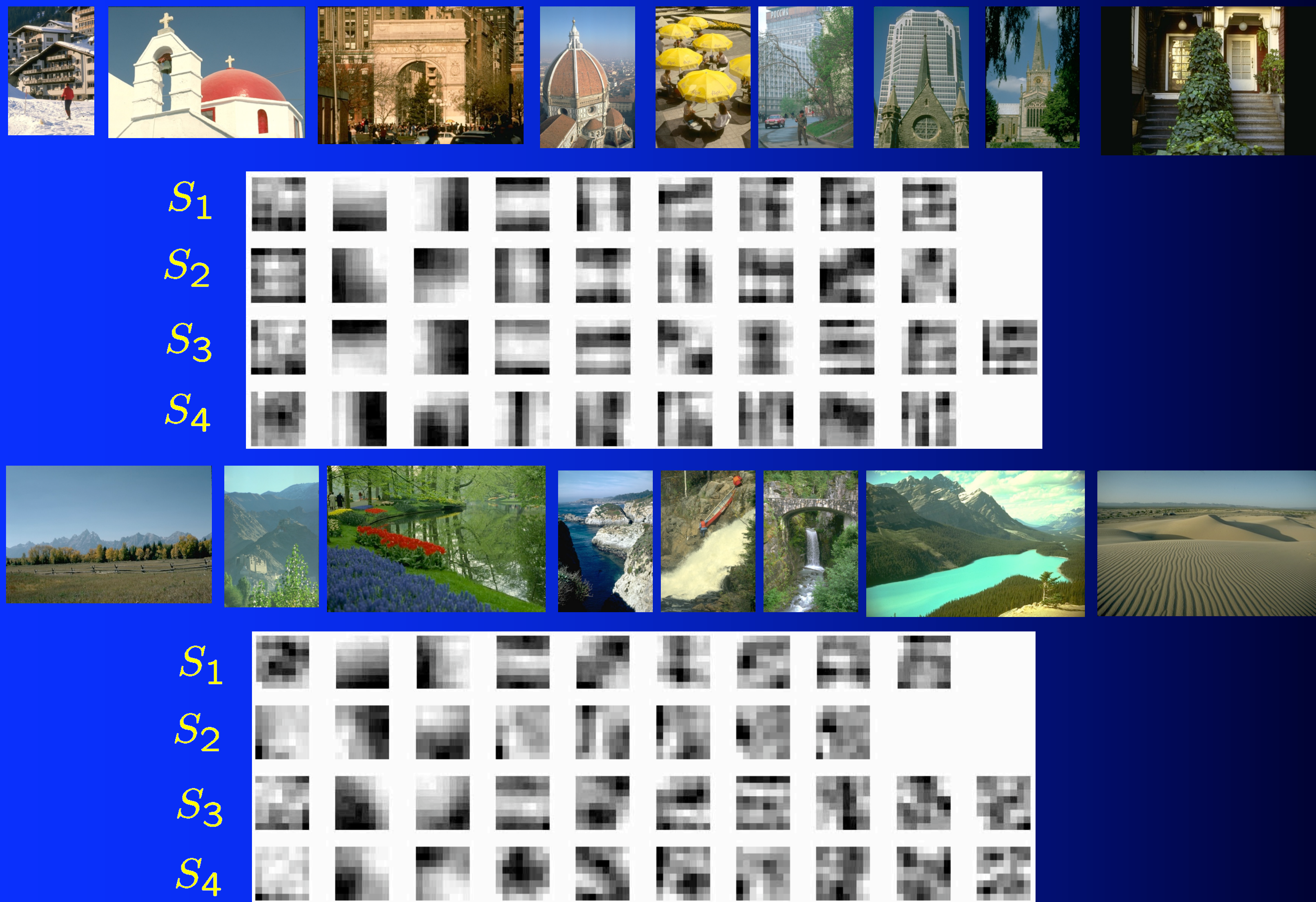
Other Applications – Image Segmentation





Other Applications – Sparse Representation of Image Ensemble



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Conclusions

- Most imagery data are high-dimensional, statistically or geometrically heterogeneous, and have multi-scale structures.
- Imagery data require hybrid models that can adaptively represent different subsets of the data with different (sparse) linear models.
- Mathematically, it is possible to estimate and segment hybrid (linear) models non-iteratively. GPCA offers one such method.
- Hybrid models lead to new paradigms, new principles, and new applications for image representation, approximation, and compression.

Future Directions

- Mathematical Theory
  - Subspace arrangements (algebraic properties).
  - Extension of GPCA to more complex algebraic varieties (e.g., hybrid multilinear, high-order tensors).
  - Representation & approximation of vector-valued functions.
- Computation & Algorithm Development
  - Efficiency, noise sensitivity, outlier elimination.
  - Other ways to combine with wavelets and curvelets.
- Applications to Other Data
  - Medical imaging (ultra-sonic, MRI, diffusion tensor…)
  - Satellite hyper-spectral imaging.
  - Audio and video.
  - Sensor networks (location, temperature, pressure, RFID…)
  - Bioinformatics (gene expression data…)

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Yi Ma

Thank You!