Mathematics of Deep Learning

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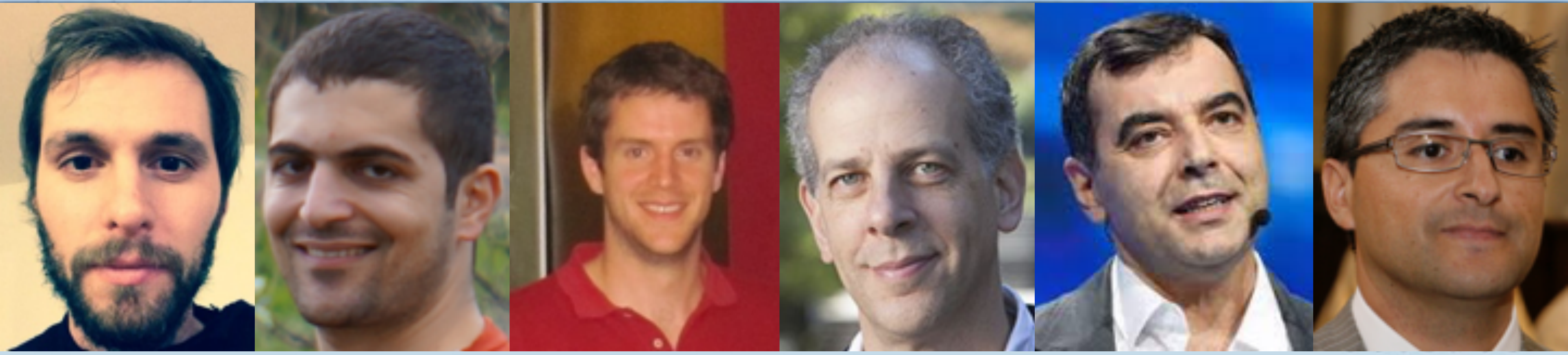
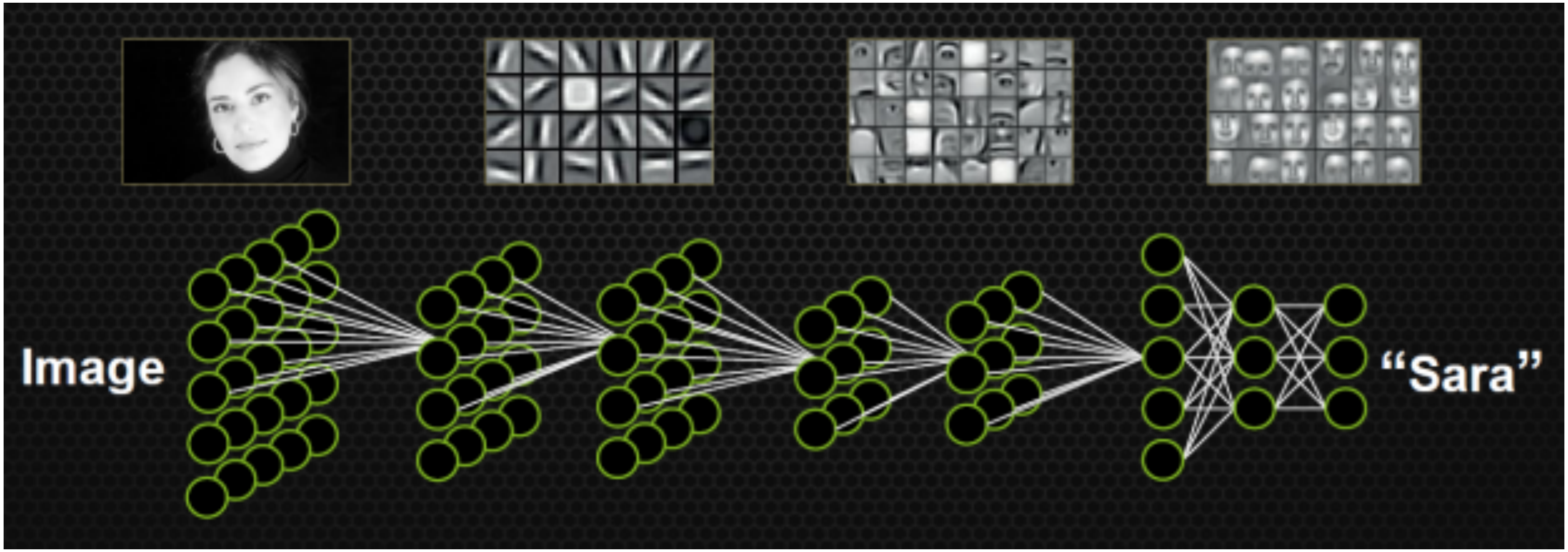


Learning Deep Image Feature Hierarchies

- Deep learning gives ~ 10% improvement on ImageNet
  - 1.2M images
  - 1000 categories
  - 60 million parameters



[1] Krizhevsky, Sutskever and Hinton. ImageNet classification with deep convolutional neural networks, NIPS’12.
[2] Sermanet, Eigen, Zhang, Mathieu, Fergus, LeCun. Overfeat: Integrated recognition, localization and detection using convolutional networks. ICLR’14.
[3] Donahue, Jia, Vinyals, Hoffman, Zhang, Tzeng, Darrell. Decaf: A deep convolutional activation feature for generic visual recognition. ICML’14.

Impact of Deep Learning in Computer Vision

- 2012-2014 classification results in ImageNet

CNN
non-CNN

2012 Teams	%error
Supervision (Toronto)	15.3
ISI (Tokyo)	26.1
VGG (Oxford)	26.9
XRCE/INRIA	27.0
UvA (Amsterdam)	29.6
INRIA/LEAR	33.4

2013 Teams	%error
Clarifai (NYU spinoff)	11.7
NUS (singapore)	12.9
Zeiler-Fergus (NYU)	13.5
A. Howard	13.5
OverFeat (NYU)	14.1
UvA (Amsterdam)	14.2
Adobe	15.2
VGG (Oxford)	15.2
VGG (Oxford)	23.0

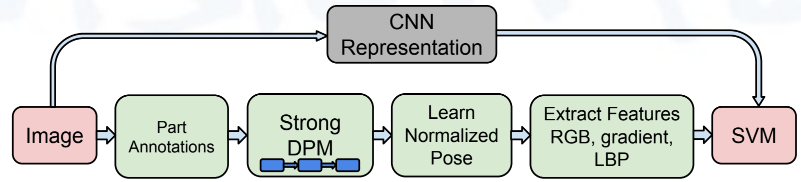
2014 Teams	%error
GoogLeNet	6.6
VGG (Oxford)	7.3
MSRA	8.0
A. Howard	8.1
DeeperVision	9.5
NUS-BST	9.7
TTIC-ECP	10.2
XYZ	11.2
UvA	12.1

- 2015 results: MSR under 3.5% error using 150 layers!

Transfer from ImageNet to Other Datasets

- ## CNNs + SMVs [1]

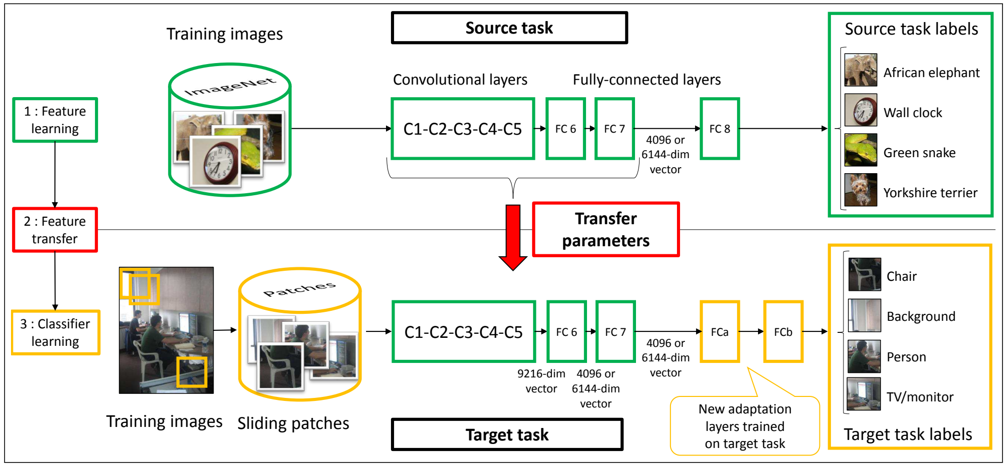
Pascal VOC 2007	mAP
GHM[8]	64.7
AGS[11]	71.1
NUS[39]	70.5
CNN-SVM	73.9
CNNaug-SVM	**77.2**



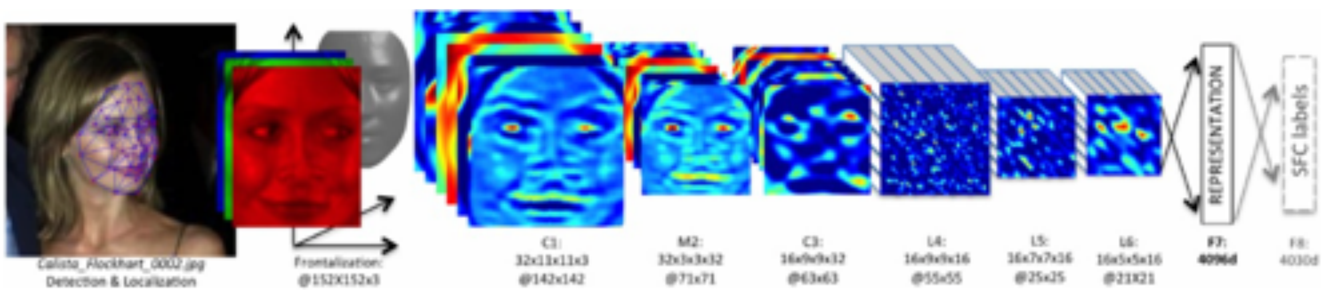
- ## Retrain top-layer [2]

Pascal VOC 2007	mAP
INRIA [32]	59.4
NUS-PSL [44]	70.5
PRE-1000C	**77.7**

Pascal VOC 2012	mAP
NUS-PSL [49]	82.2
NO PRETRAIN	70.9
PRE-1000C	78.7
PRE-1000R	76.3
PRE-1512	**82.8**



- ## Deep Face [3]

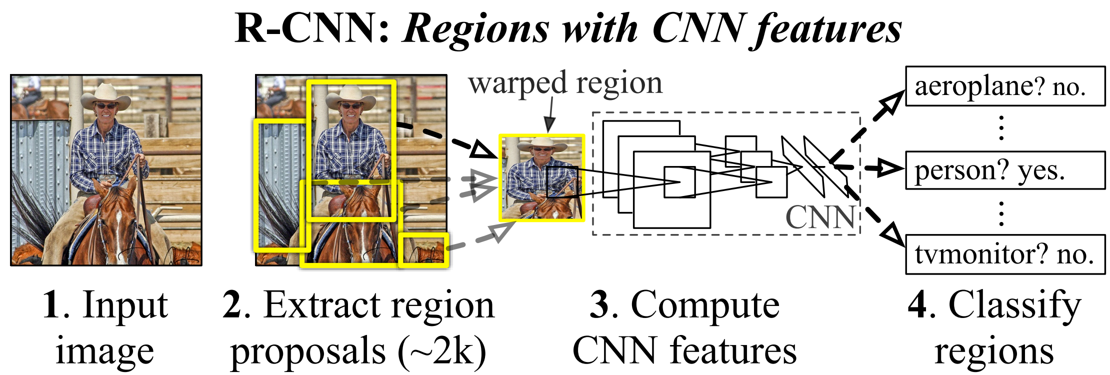


[1] Razavian, Azizpour, Sullivan, Carlsson, CNN Features off-the-shelf: an Astounding Baseline for Recognition. CVPRW'14.
[2] Oquab, Bottou, Laptev, Sivic. Learning and transferring mid-level image representations using convolutional neural networks CVPR'14
[3] Taigman, Yang, Ranzato, Wolf. DeepFace: Closing the Gap to Human-Level Performance in Face Verification. CVPR'14

Transfer from Classification to Other Tasks

- CNNs + SVMs for object detection [1,2]

VOC 2010 test	mAP
DPM v5 [20]†	33.4
UVA [39]	35.1
Regionlets [41]	39.7
SegDPM [18]†	40.4
R-CNN	50.2
R-CNN BB	**53.7**

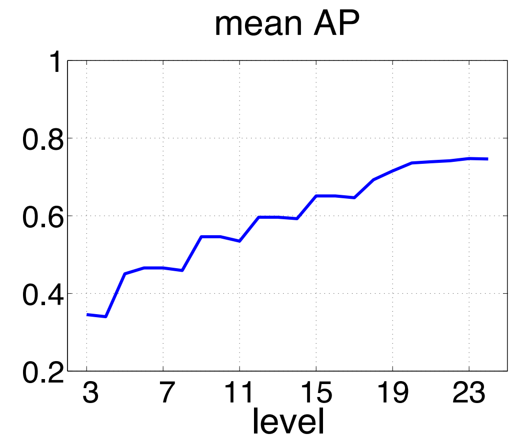


- CNNs for pose estimation [3] and semantic segmentation [4]

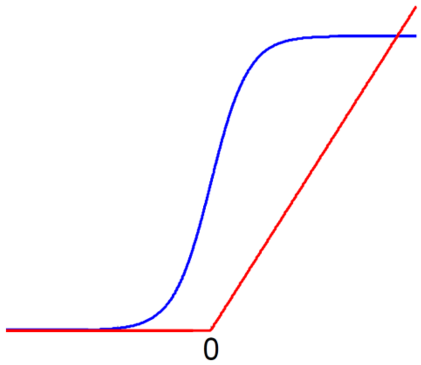
[1] Girshick, Donahue, Darrell and Malik. Rich feature hierarchies for accurate object detection and semantic segmentation, CVPR'14
[2] Sermanet, Eigen, Zhang, Mathieu, Fergus, LeCun. OverFeat: Integrated Recognition, Localization and Detection using Convolutional Networks. ICLR
[3] Tompson, Goroshin, Jain, LeCun, Bregler. Efficient Object Localization Using Convolutional Networks. CVPR'15
[4] Pinheiro, Collobert, Dollar. Learning to Segment Object Candidates. NIPS'15

Why These Improvements in Performance?

- Features are learned rather than hand-crafted
- More layers capture more invariances [1]



- More data to train deeper networks
- More computing (GPUs)
- Better regularization: Dropout
- New nonlinearities
  - Max pooling, Rectified linear units (ReLU)



- Theoretical understanding of deep networks remains shallow

[1] Razavian, Azizpour, Sullivan, Carlsson, CNN Features off-the-shelf: an Astounding Baseline for Recognition. CVPRW'14.

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Early Theoretical Results on Deep Learning

- **Approximation theory**
  - Perceptrons and multilayer feedforward networks are universal approximators: Cybenko ’89, Hornik ’89, Hornik ’91, Barron ‘93

- **Optimization theory**
  - No spurious local optima for linear networks: Baldi & Hornik ’89
  - Stuck in local minima: Brady ‘89
  - Stuck in local minima, but convergence guarantees for linearly separable data: Gori & Tesi ‘92
  - Manifold of spurious local optima: Frasconi ’97

[1] Cybenko. Approximations by superpositions of sigmoidal functions, Mathematics of Control, Signals, and Systems, 2 (4), 303-314, 1989.
[2] Hornik, Stinchcombe and White. Multilayer feedforward networks are universal approximators, Neural Networks, 2(3), 359-366, 1989.
[3] Hornik. Approximation Capabilities of Multilayer Feedforward Networks, Neural Networks, 4(2), 251–257, 1991.
[4] Barron. Universal approximation bounds for superpositions of a sigmoidal function. IEEE Transactions on Information Theory, 39(3):930–945, 1993.

[5] P Baldi, K Hornik, Neural networks and principal component analysis: Learning from examples without local minima, Neural networks, 1989.
[6] Brady, Raghavan, Slawny. Back propagation fails to separate where perceptrons succeed. IEEE Trans Circuits & Systems, 36(5):665–674, 1989.
[7] Gori, Tesi. On the problem of local minima in backpropagation. IEEE Trans. on Pattern Analysis and Machine Intelligence, 14(1):76–86, 1992.
[8] Frasconi, Gori, Tesi. Successes and failures of backpropagation: A theoretical. Progress in Neural Networks: Architecture, 5:205, 1997.

Recent Theoretical Results on Deep Learning

- **Invariance, stability, and learning theory**
  - Scattering networks: Bruna ’11, Bruna ’13, Mallat ’13
  - Deformation stability for Lipschitz non-linearities: Wiatowski ’15
  - Distance and margin-preserving embeddings: Giryes ’15, Sokolik ‘16
  - Geometry, generalization bounds and depth efficiency: Montufar ’15, Neyshabur ’15, Shashua ’14 ’15 ‘16
- **Optimization theory and algorithms**
  - Learning low-degree polynomials from random initialization: Andoni‘14
  - Characterizing loss surface and attacking the saddle point problem: Dauphin ‘14 , Choromanska ’15, Chaudhuri ‘15
  - Global optimality in neural network training: Haeffele ‘15
  - Training NNs using tensor methods: Janzamin ’15

[1] Bruna-Mallat. Classification with scattering operators, CVPR’11. Invariant scattering convolution networks, arXiv’12. Mallat-Waldspurger. Deep Learning by Scattering, arXiv’13.
[2] Wiatowski, Bölcskei. A mathematical theory of deep convolutional neural networks for feature extraction. arXiv 2015.
[3] Giryes, Sapiro, A Bronstein. Deep Neural Networks with Random Gaussian Weights: A Universal Classification Strategy? arXiv:1504.08291.
[4] Sokolic. Margin Preservation of Deep Neural Networks, 2015
[5] Montufar. Geometric and Combinatorial Perspectives on Deep Neural Networks, 2015.
[6] Neyshabur. The Geometry of Optimization and Generalization in Neural Networks: A Path-based Approach, 2015.
[7] Andoni, Panigraphy, Valiant, Zhang. Learning Polynomials with Neural Networks. ICML 2014.
[8] Dauphin, Pascanu, Gulcehre, Cho, Ganguli, Bengio, Identifying and attacking the saddle point problem in high-dimensional non-convex optimization. NIPS 2014.
[9] Choromanska, Henaff, Mathieu, Arous, LeCun, “The Loss Surfaces of Multilayer Networks,” AISTAT 2015.
[10] Chaudhuri and Soatto The Effect of Gradient Noise on the Energy Landscape of Deep Networks, arXiV 2015.
[11] Haeffele, Vidal. Global Optimality in Tensor Factorization, Deep Learning and Beyond, arXiv, 2015.
[12] Janzamin, Sedghi, Anandkumar, Beating the Perils of Non-Convexity: Guaranteed Training of Neural Networks using Tensor Methods, arxiv 2015.

Motivations and Goals of this Tutorial

- **Motivation:** Deep networks have led to dramatic improvements in performance for many tasks, but the mathematical reasons for this success remain unclear.

- **Goal:** Review very recent work that aims at understanding the mathematical reasons for the success of deep networks.

- **What we will do:** Study theoretical questions such as
  - What properties of images are being captured/exploited by DNNs?
  - Can we ensure that the learned representations are globally optimal?
  - Can we ensure that the learned representations are stable?

- **What we will not do:** Show X% improvement in performance for a particular application.

Tutorial Schedule

- 14:00-14:15: **René Vidal** - Introduction
- 14:15-15:00: **Amnon Shashua** - On Depth Efficiency of Convolutional Networks: Theory and Implications for Practical Architectures
- 15:00-15:45: **René Vidal and Benjamin Haeffele** - Global Optimality and Regularization in Deep Learning
- 15:45-16:15: Coffee Break
- 16:15-17:00: **Raja Giryes and Guillermo Shapiro** - Data Structure Based Theory of Deep Learning
- 17:00-17:45: **Joan Bruna** - Addressing Curse of Dimensionality with Convolutional Neural Networks
- 17:45-18:00: Discussion